Research plan

Eliashberg said that contact topology is the true heir (successor) of low dimensional topology. In view of this, I wish to research in your institute into the next three subjects which require rich experience in studying low dimensional cases:

i) Contact embeddings and singularity theory: Martínez Torres, generalizing my result, constructed a contact immersion of a given closed contact $(2n+1)$-manifold M^{2n+1} into the standard $(4n+1)$-sphere $S^{4n+1} \subset \mathbb{C}^{2n+1}$ such that the pull-back of the trivial open-book decomposition of S^{4n+1} is a symplectic open-book decomposition of M^{2n+1}. On the other hand, since M^{2n+1} is oriented, we can embed M^{2n+1} into S^{4n+1} as a smooth submanifold. I am now trying to show that the above contact immersion can be taken as a contact embedding. Indeed I can so embed 'almost all' contact 3-manifolds into S^5. I would like to extend the notion of closed braid so as to contain "spinning" contact submanifolds. Here I aim to apply it to the study on complex singularities, especially on surface singularities.

ii) Leafwise symplectic foliations: An almost contact structure on M^{2n+1} is a G-structure by the 1-jets of local contact transformations of $(J^1(n,1),0)$. We may regard it as a pair $([\alpha],[\omega])$ of conformal classes of a 1-form α and a 2-form ω which satisfies $[\alpha] \wedge [\omega]^n > 0$. A contact form α determines an almost contact structure just by putting $\omega = d\alpha$. Then the almost contact structure is said to be exact. Another typical example of an almost contact structure is a leafwise almost symplectic structure of a (codimension one) foliation. Recently, motivated by an effort of Verjovsky et al., Mitsumatsu constructed a leafwise symplectic structure on the Lawson foliation on S^5. In [11], I improved and generalized his result. Namely, I constructed a family of almost contact structures on each contact manifold of a certain class which starts with the exact one and ends with a leafwise symplectic structure with keeping $\ker[\alpha]$ contact. This result leads us to generalize the Eliashberg-Thurston theory. I also try to generalize the Novikov closed leaf theorem to leafwise symplectic case. Note that the recent work of Meigniez implies that high-dimensional Novikov theorem never hold in smooth case.

iii) Submanifolds foliated by Legendrian submanifolds: A Legendrian submanifold L of a contact $(4n+3)$-manifold is a $(2n+1)$-manifold. Since a neighbourhood of the 0-section of the cotangent bundle T^*L is also embedded, we can perturb L to obtain a contact submanifold L' as long as L admits a contact structure. (Thus L can not control the contact nature of L'.) On the other hand, a certain 1-dimensional family of Legendrian submanifolds of a contact $(4n+1)$-manifold forms a foliation, and does control a family of contact structures which converges to the foliation. This phenomenon was first found by Bennequin. For most of my results somehow related to this phenomenon, I would like to further persuit it.