Research outline

A contact $(2n+1)$-manifold M^{2n+1} is locally modelled on the 1-jet space $J^1(n,1)$ for a function with n variables. Precisely, given a Γ-structure on M^{2n+1} by the pseudogroup Γ of local contact transformations of $J^1(n,1)$, we call its invariant hyperplane distribution D a contact structure on M^{2n+1}. Usually we assume D to be suitably co-oriented, and take a global contact 1-form α with $D = \ker\alpha$. Then we can take an atlas consisting of charts sending α to the canonical form of $J^1(n,1)$ (Darboux's theorem). A contact structure arises as a transverse invariant structure of a dilative flow on a symplectic manifold, and each contact form as a section of the flow. In the case where M^{2n+1} is closed, any deformation of D is trivial (Gray stability). Particularly, contact structures are isotopic if they are homotopic. That is why contact geometry has a topological nature.

Thurston and Winkelnkemper constructed a contact structure on a given (closed oriented) 3-manifold equipped with an open-book decomposition. In [3] I showed that it comes from a symplectic filling if the monodromy is 'right-handed'. Loi and Piergallini showed that a 3-manifold is diffeomorphic to the boundary of a Stein domain iff it admits a 'right-handed' open-book. These results are later unified and included in Giroux's one-to-one correspondance between contact structures and positive stabilization classes of open-books. I also showed that any contact structure on M^3 can be deformed into a spinnable foliation. This implies that the relative Thurston inequality holds for many foliations with Reeb components in contrast to the Eliashberg-Thurston theory. With collaborators, I studied on this phenomena and obtained some results: See [7] for homological overtwistedness, [6] for Dehn fillings, and [5] for a generalization of Bennequin's isotopy lemma.

In [4] and [13] I constructed a certain immersion (sometimes embedding) of a given contact M^3 to $J^1(2,1)$ by using approximately holomorphic geometry. This result has been generalized by Martínez Torres. In [8] I isotoped the 'standard' 3-sphere in $J^1(2,1)$ through contact submanifolds to a union of Legendrian submanifolds forming the Reeb foliation, and then to an 'exotic'(=overtwisted) contact 3-sphere. Here the Reeb foliation is presented by a 1st order SPDE.

Any Seifert surface in $J^1(1,1) \approx S^3 \setminus \{*\}$ satisfies Bennequin's inequality, and any surface in a contact 3-manifold is smoothly approximated by a 'convex' surface. In [10] I constructed a Seifert hypersurface in $J^1(2,1) \approx S^5 \setminus \{*\}$ which violates the inequality and is far from 'convex'. Lutz modified the contact structure of $J^1(1,1)$ into exotic one. In [9], using geometry of Brieskorn 3-manifolds, I generalized this modification to $J^2(2,1)$. Here I obtained a 'convex' Seifert hypersurface which violates the inequality and obstracts symplectic fillability. This result has been partially generalized by Massot, Niederkrüger and Wendl.

I also have a collaboration [1] with Fukui on (in)stability of certain foliations.