Research Plan

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The curve complex $\mathcal{C}(S)$ of a Riemann surface S was introduced to study the geometry of the Teichmülller space $T(S)$, in particular to encode the asymptotic geometry of $T(S)$ by Harvey.

The important theorem of the curve complex is Masur-Minsky's result that the curve complex is hyperbolic in the sense of Gromov. Then the curve complex is, nowadays, recognized as important tool in the theory of Teichmülller space

Our research purpose is to *analyze Teichmüller spaces by using the theory of curve complex.*

In the near future, we study the following problems ;

(1) A non-extendibility of the map $\widehat{\varphi} : T(S) \times U \to T(\dot{S})$ to $T(S) \times (U \cup (\partial U - \mathbb{A}))$

We have found the extendibility of the map $\widehat{\varphi}$ to $T(S) \times (U \cup \mathbb{A})$. Thus next I will consider a non-extendibility of $\widehat{\varphi}$ to $T(S) \times (\partial U - \mathbb{A})$. Now I got to find that $T(S) \times (\partial U - \mathbb{A})$ is too "large". Hence, to prove the non-extendibility, we need some assumptions.

Our strategy is to generalize Zhang's proof in "*Non-extendibility of the Bers isomorphism*". The key is to study the set A of accumulation points of the iteration of an element in the Teichmüller modular group.

Zhang investigates A in the case of an element be parabolic and we can apply many classic results to it. Though we need to do A in the case of an element be pseudo-hyperbolic, I do not have a useful tool. We expect to apply some arguments in Brock's Ph D thesis to A

(2) An extendibility of the map $\widehat{\varphi} : T(S) \times U \to T(\dot{S})$ to $\overline{T(S)}^B \times U$

I will consider an extendibility of $\widehat{\varphi} : T(S) \times U \to T(\dot{S})$ to $\overline{T(S)}^B \times U \to \overline{T(\dot{S})}^B$, where $\overline{T}^B$ is the Bers' compactification of T.

However I think this problem is difficult. So first, I try to consider an extendibility of the map $\widehat{\varphi}$ to $(T(S) \cup \mathbb{P}FL(S)) \times U \to (T(\dot{S}) \cup \mathbb{P}FL(\dot{S}))$.

To do this, we use two results as follows;

(i) a *natural* map from $T \to \mathcal{C}$ sending X to any shortest curve in X extends to $(T \cup \mathbb{P}FL) \to \overline{\mathcal{C}}$ continuously. (E. Klarreich, *The Boundary at infinity of the curve complex and relative Teichmüller Spaces*, Theorem 1.1).

(ii) the map $\Phi : \mathcal{C}(S) \times U \to \mathcal{C}(\dot{S})$ extends to $\overline{\mathcal{C}}(S) \times U \to \overline{\mathcal{C}}(\dot{S})$ continuously (C. J. Leininger, M. Mj and S. Schleimer, *The universal Cannon-Thurston map and the boundary of the curve complex,* Proposition 2.11).

In accordance with observations as above, we try the original problem.