Research Results

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On holomorphic family or Riemann surfaces

A family $\{S_b\}_{b\in B}$ of Riemann surfaces over a Riemann surface B whose S_b depends holomorphically on $b \in B$ is called a *holomorphic family of Riemann surfaces over* B. For a given holomorphic family, it is a fundamental problem to estimate the number of holomorphic sections of it. Here, a holomorphic map s from B into $\{S_b\}_{b\in B}$ is said to be a *holomorphic section* of it if the composed map of s and the projection $\{S_b\}_{b\in B} \to B$ is the identity map of B. In general, it is difficult to estimate the number.

In [1] and [2], we consider sections of a holomorphic family over B in the case where B is a torus with four punctures. Our strategy is to estimate the number of holomorphic maps between tori induced from the family instead of estimating the number of sections. The holomorphic map can be lifted to the universal coverings (the complex plane) of the tori and it has concrete form. By using the form, we can estimate the number of the maps and then we see that there are at most ten sections of our family.

However we can not apply the argument as above to the case where B is a closed Riemann surface of genus $g \geqq 2$. To generalize the argument is the next problem.

On the extendibility of the Bers isomorphism

Bers constructed the isomorphism φ of the Bers fiber space $F(S)$ of a closed Riemann surface S of genus $g(\geqq 2)$ onto the Teichmülller space $T(\dot{S})$ of the Riemann surface $\dot{S} = S - \{\hat{a}\}$ with one puncture. It is called the *Bers isomorphism*. Since $F(S)$ and $T(\dot{S})$ are represented as bounded domains in $\mathbb{C}^{3g-2}$ respectively, the topological boundaries of them are naturally defined.

We have the following problem: Whether the Bers isomorphism φ of $F(S)$ onto $T(\dot{S})$ has a continuous extension to the closure $\overline{F(S)}$ of $F(S)$? Zhang proved that φ cannot be continuously extended to $\overline{F(S)}$ if the dimension of $T(S)$ is greater than zero. Then we have the next question: Is there a subset of the boundary $\overline{F(S)} - F(S)$ to which φ can be continuously extended ?

In this research, instead of φ, we take a composed map $\varphi \circ r : T(S) \times U \to T(\dot{S})$ of φ and a real analytic map $r : T(S) \times U \to$ (U : the upper half plane) and then consider the above question. Denote it by $\widehat{\varphi}$. Let $\mathbb{A} \subset \partial U$ be a subset of all points filling S, where ∂U is the boundary of U. By a joint work with H. Miyach (Osaka University), we found that $\widehat{\varphi}$ can be continuously extended to $T(S) \times (U \cup \mathbb{A}) \to \overline{T(\dot{S})}^{B}$.