results of research

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A knot is the image of an embedding of circle in the 3-sphere S^3, denoted by K. A link is the image of an embedding of circles $S^1 \cup S^1 \cup \cdots \cup S^1$ in the 3-sphere S^3. Let $L = K_1 \cup K_2 \cup \cdots \cup K_n$ be an n-component link in S^3, and $N(L)$ a tubular neighborhood of L, and $E(L)$ the exterior of L. Let $\chi(L, 0)$ be the 3-manifold obtained from $E(L)$ by attaching n solid tori $V_1, V_2, \ldots, V_n$ to $\partial E(L)$ such that the meridian of ∂V_i is mapped to the longitude of K_i $(i = 1, 2, \ldots, n)$. We call $\chi(L, 0)$ the *3-manifold obtained by the 0-surgery of* S^3 along L. It is well known that every closed connected orientable 3-manifold is obtained by the 0-surgery of S^3 along a link.

Let M be a closed connected orientable 3-manifold. For any M, there exist handlebodies H_1 and H_2 of same genus and a homeomorphism $f : \partial H_1 \to \partial H_2$ such that $M = H_1 \cup_f H_2$. We call the triple $(H_1, H_2; f)$ a *Heegaard splitting* of M and we call $f(\partial H_1) = \partial H_2$ the *Heegaard surface*. The *Heegaard genus* of M is the minimal genus of Heegaard surfaces, denoted by $g_{\mathrm{H}}(M)$.

Let bridge(L) (resp. braid(L)) be the bridge index (resp. the braid index) (cf. [5]). The *bridge genus* $g_{\mathrm{bridge}}(M)$ (resp. the *braid genus* $g_{\mathrm{braid}}(M)$) of M is the minimal number of bridge(L) (resp. braid(L)) for any L such that M is obtained by the 0-surgery of S^3 along L. The bridge genus and the braid genus are introduced by A.Kawauchi [6].

I show the following results and my paper *On Heegaard genus, bridge genus and braid genus for a 3-manifold* is published in Journal of Knot Theory and Its Ramifications.

$$g_{\mathrm{H}}(M) \leq g_{\mathrm{bridge}}(M) \leq g_{\mathrm{braid}}(M).$$

There exist 3-manifolds which satisfy each one of the inequalities.

$$g_{\mathrm{H}}(M) = g_{\mathrm{bridge}}(M) = g_{\mathrm{braid}}(M), \quad (1)$$
$$g_{\mathrm{H}}(M) < g_{\mathrm{bridge}}(M) = g_{\mathrm{braid}}(M), \quad (2)$$
$$g_{\mathrm{H}}(M) = g_{\mathrm{bridge}}(M) < g_{\mathrm{braid}}(M), \quad (3)$$
$$g_{\mathrm{H}}(M) < g_{\mathrm{bridge}}(M) < g_{\mathrm{braid}}(M). \quad (4)$$

If there exist complex numbers a, b and c which satisfy $ag_{\mathrm{H}}(M)+bg_{\mathrm{bridge}}(M)+cg_{\mathrm{braid}}(M) = 0$ for all 3-manifolds M, then $a = b = c = 0$. Thus, the invariants g_{H}, g_{bridge} and g_{braid} are linearly independent.