Results

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The problem to construct a distinguished metrics on a manifolds is important in differential geometry. The Einstein structures and the Ricci soliton structures are candidates to this question. I study the Ricci soliton structures.

If g_0 satisfies

$$\mathrm{Ric}[g_0] = cg_0 + L_X g_0 \ ,$$

where Ric is the Ricci tensor of g_0, X is a vector field and c is a constant, then (M^n, g_0, X, α) is called a *Ricci soliton structure* and g_0 *the Ricci soliton.* A Ricci soliton is a Ricci flow solution $\frac{\partial}{\partial t} g(t)_{ij} = -2Rc[g(t)]_{ij}$.

In general, problems for Ricci solitons are second-order differential equations. Lauret introduced sol-solitons in Riemannian case.

Definition 1. *Let* (G, g) *be a simply connected Lie group equipped with the left-invariant pseudo-Riemannian metric* g*, and let* $\mathfrak{g}$ *denote the Lie algebra of* G*. Then* g *is called an algebraic Ricci soliton if it satisfies*

$$\mathrm{Ric} = c\mathrm{Id} + D \tag{1}$$

where Ric *denotes the Ricci operator,* c *is a real number, and* $\mathrm{D} \in \mathrm{Der}\,(\mathfrak{g})$ *(*D *is a derivation of* $\mathfrak{g}$*), that is:*$\mathrm{D}[X, Y] = [\mathrm{D}X, Y] + [X, \mathrm{D}Y]$ *for any* $X, Y \in \mathfrak{g}$*. In particular, an algebraic Ricci soliton on a solvable Lie group, (a nilpotent Lie group) is called a sol-soliton (a nil-soliton).*

Lauret proved that sol-solitons on homogeneous Riemannian manifolds are Ricci solitons. Algebraic soliton (Sol-solitons) allow us to construct Ricci solitons in an algebraic way, i.e., using algebraic soliton (sol-soliton) theory, the study of Ricci solitons on homogeneous manifolds becomes algebraic.

I studied sol-solitons on solvable Lie groups in the Lorentzian case ([3]). By using the sol-soliton theory, we can construct homogeneous Lorentzian Ricci solitons in an algebraic way. I constructed Lorentzian sol-solitons on H_3, $E(2)$, $E(1,1)$, H_N, and $G_m(\lambda)$. I obtain the shrinking sol-soliton on H_3, $E(2)$, $E(1,1)$, H_N, and the steady sol-soliton on $G_m(\lambda)$ without Riemannian analog. In [4], I gave the complete classification of algebraic Ricci soliton in three-dimensional unimodular Lie groups and in non-symmetric non-unimodular Lie groups. In [5], I completely classify the algebraic Ricci solitons of four-dimensional pseudo-Riemannian generalized symmetric spaces.