Results of my research

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I have studied the following nonlinear Schrödinger equations:

$$-\epsilon^2\Delta u + V(x)u = f(u) \quad \text{in } \mathbf{R}^N \qquad (*)_\epsilon$$

In particular, I mainly studied perturbation problem for $(*)_\epsilon$, that is, I resarched the multiplicites of solutions of $(*)_\epsilon$ and shapes of the those solutions when $\epsilon \to 0$. It is well-known that $(*)_\epsilon$ has positive solutions which has local muximum points around one (or some) critical points of non-negative function $V(x)$ and closed to 0 on the other points. In what follows, we call solution which has only one (more than 2) maximum points single-peak (multi-peak) solutions, respectively.

These peak solutions have the following properties:

1. The locations of maximum point of peak solution must close to critical point of $V(x)$.

2. The multi-peak solutions has similar shapes to single-peak solutions around each critical points of $V(x)$.

Thus it is a fundamental question that whether we can construct a solution which has peaks around prescribed one or some critical points of $V(x)$. In other ward, it is that whether we can consruct a solution joining prescribed single-peak solutions.

One of the feature of our researches is that we dealt with a solution joining different type of single-peak solutions each other. In what follows, we state about my papers ([1]∼[5]) briefly.

• **Research in [1].** Byeon-Wang showed that the shapes of single-peak solutions are drastically different whether a local minimum value of $V(x)$ is positive or 0. In fact, for the single-peak solutions which has maximum point around x_0 (x_0 is a local minimum point of $V(x)$ with $V(x_0) = 0$.), they observed that the shapes of such solutions depend on the way of the degeneration of $V(x)$ at $x = x_0$.

In [1], we assumed that $f(u) = |u|^{p-1}u$ and constructed multi-peak solutions which has peaks around descrived local minimum points of $V(x)$.

• **Research in [2].** Whang observed that, when $V(x) > 0$, $(*)_\epsilon$ does not have positive multi-peak solutions whose peaks constructed around one same local minimum point of $V(x)$. On the other hand, Alves–Soares showed that, when $V(x) > 0$, $(*)_\epsilon$ has two-peak solutions whose one peak is positive and another peak is negative and both peaks constructed around one same local minimum point of $V(x)$. In [2], we constructed sign-changing two-peak solutions around x_0 where x_0 is a local minimum point of $V(x)$ with $V(x_0) = 0$. Moreover, our proof covers gap of arguments in papar of Alves–Soares.

• **Research in [3].** This is a joint work with Professor K. Tanaka. In this papar, we consider the following type of nonlinear Schrödinger equations:

$$-\Delta u + (\lambda^2 a(x) + 1)u = |u|^{p-1}u \quad \text{in } \mathbf{R}^N, \quad u \in H^1(\mathbf{R}^N). \qquad (*)_\lambda$$

In [3], we assume that the bottom of $V(x) = \lambda^2 a(x) + 1$ consists two connected components, that is, $\Omega_1 \cup \Omega_2 = \{x \in \mathbf{R}^N;\, a(x) = 0\}$ ($\Omega_1 \cap \Omega_2 = \emptyset$). In our situation, letting u_λ be a family of solutions whose H^1-norm bounded, after extracting a subsequence, u_{λ_j} approaches to 0 outside of $\Omega_1 \cup \Omega_2$ and converges to a solution of Dirichelt problems $(L)_{\Omega_i}$ $-\Delta u + u = |u|^{p-1}u$ in Ω_i ($i = 1, 2$). On the other hand, it is well-known that $(L)_{\Omega_i}$ has infinitly many solutions from the symmetric mountain pass theory.

Thus, in [3], we consider the problems that whether we can construct multi-peak solutions of $(*)_\lambda$ joining a prescribed pair of solutions of $(L)_{\Omega_i}$ ($i = 1, 2$). And we got partial answer of this question. That is, we showed that there exists solutions of $(*)_\lambda$ joining a least energy solution of $(L)_{\Omega_1}$ and a solution of $(L)_{\Omega_2}$ correponding to critical values defiend by a symmetric mountain pass theory. Moreover, the number of such solutions goes to infinity as $\lambda \to \infty$.