Research Planning

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1 Research on the incompressible Euler equations

1.1 Well-posedness and ill-posedness of the initial value problem

We consider the well-posedness and the ill-posedness for the initial value problem of the incompressible Euler equations. In particular, we investigate the border of function spaces between the well-posedness and the ill-posedness in the framework of the Besov spaces and the Triebel-Lizorkin spaces. Concerning the well-posedness, it is known that the Euler equation is locally well-posed in the Besov spaces $B^1_{\infty,1}(\mathbb{R}^n)$. On the other hand, we proved in [A3] that the nonlinear estimates fail in $B^1_{\infty,q}(\mathbb{R}^n)$ provided $1 < q \leqslant \infty$. Hence we aim to show that the Euler equation is ill-posed in $B^1_{\infty,q}(\mathbb{R}^n)$ with $1 < q \leqslant \infty$. We also consider this problem in the Triebel-Lizorkin spaces.

1.2 Blow-up criterion for the local in time solutions

We aim to make some improvements for the Beale-Kato-Majda blow-up criterion for the local in time solutions to the 3-dimensional incompressible Euler equations. In the case of the Navier-Stokes equations, it is known that the solution on $(0,T)$ can be extended beyond T if the two components of the vorticity belong to $L^1(0,T;\mathrm{BMO}(\mathbb{R}^3))$. On the other hand, in the case of the Euler equations, the corresponding result is known only in the class $L^2(0,T;\dot{B}^0_{\infty,1}(\mathbb{R}^3))$, which is strictly included in $L^1(0,T;\mathrm{BMO}(\mathbb{R}^3))$ and is not scaling invariant space. Hence our aim is to prove the blow-up criterion via two components of the vorticity for the Euler equations in some scaling invariant space like $L^1(0,T;\mathrm{BMO}(\mathbb{R}^3))$.

2 Research on the Navier-Stokes equations with the Coriolis force

2.1 Dispersive effect of the Coriolis force and the global well-posedness

We consider the global well-posedness for the initial value problem of the Navier-Stokes equations with the Coriolis force. In our previous work [D1], we proved the global well-posedness for small initial data, where the condition on the initial data is independent of the Coriolis parameter. We aim to make it clear the relation between the size of initial data for the global well-posedness and the Coriolis parameter, and prove the global well-posedness for large initial data provided the speed of rotation is sufficiently fast.

Furthermore, we also consider the asymptotic behavior of solutions and the problem of fast singular oscillating limits as the speed of rotation tends to infinity.

2.2 Dispersive effect of the Coriolis force and the stationary problem

We consider the stationary Navier-Stokes equations in the rotational framework. It is known that if the stationary external force is sufficiently and uniformly small with respect to the Coriolis parameter, then there exists a stationary solution. We aim to make it clear the relation between the size of the external force for the existence of stationary solutions and the Coriolis parameter, and prove the existence of stationary solutions for large external force when the speed of rotation is sufficiently fast. We also consider the case that the external force is periodic in time. Furthermore, we investigate the asymptotic stability for both the stationary solution and the time periodic solution.