Research Briefing Report

Ryo Takada

1 Research on the incompressible Euler equations

We have studied the well-posedness for the incompressible Euler equations, describing the motion of perfect incompressible fluids. Making use of the technique in the harmonic analysis, we proved the following three subjects. In what follows, we shall explain these subjects, respectively.

1.1 Local well-posedness of the Euler equations in Besov spaces of weak type

In [A1], we introduced Besov type function spaces, based on the weak L^p-spaces instead of the standard L^p-spaces, and proved a local in time unique existence of solutions and a blow-up criterion of Beale-Kato-Majda type in those function spaces. Our results in [A1] include the corresponding results in the usual Besov spaces.

1.2 Optimal function spaces for the well-posedness

In [A3], we investigated the optimal function spaces for the well-posedness of the Euler equations in the framework of the Besov spaces $B^s_{p,q}(\mathbb{R}^n)$ and the Triebel-Lizorkin spaces $F^s_{p,q}(\mathbb{R}^n)$. In particular, we considered this problem from a viewpoint of the commutator estimates of Kato-Ponce type, and gave a necessary and sufficient condition for the exponents s, p and q that the commutator estimates fail.

1.3 Propagation of the real analyticity

In [A4] [B1], we proved that if the initial velocity in the Besov space $B^1_{\infty,1}(\mathbb{R}^n)$ is real analytic then the solution in the class $C([0,T]; B^1_{\infty,1}(\mathbb{R}^n))$ is also real analytic in spatial variables. Furthermore, we established a estimate for the size of the radius of convergence of the Taylor expansion, and characterized the relation between the blow-up time and the radius of the convergence in terms of the Beale-Kato-Majda blow-up criterion.

2 Research on the Keller-Segel systems

We have studied the behavior of blow-up solutions to the Keller-Segel systems describing the aggregation process of amoebae chemotaxis.

In [A2], we considered the problem whether there does exist a finite-time self-similar solution of the backward type to the Keller-Segel system in $\mathbb{R}^n$. In the case of parabolic–elliptic type for $n \geqslant 3$, we show that there is no such a solution with a finite mass in the scaling invariant class. In the case of parabolic–parabolic type for $n \geqslant 2$, nonexistence of finite-time self-similar solutions is proved in a larger class of a finite mass with some local bounds.

3 Research on the Navier-Stokes equations with the Coriolis force

We have studied the well-posedness for the incompressible Navier-Stokes equations with the Coriolis force.

3.1 Global well-posedness and ill-posedness

In [D1], we introduced function spaces $\dot{\mathcal{B}}^s_{p,q}(\mathbb{R}^3)$ of Besov type, and proved the global in time existence and the uniqueness of the mild solution for small initial data in our space $\dot{\mathcal{B}}^{-1}_{1,2}(\mathbb{R}^3)$. Furthermore, we also discuss the ill-posedness in $\dot{\mathcal{B}}^{-1}_{1,q}(\mathbb{R}^3)$ with $2 < q \leqslant \infty$, which implies the optimality of our function space $\dot{\mathcal{B}}^{-1}_{1,2}(\mathbb{R}^3)$ for the global-wellposedness.

3.2 Dispersive effect of the Coriolis force and the local well-posedness

In [D2], we proved the local in time existence and uniqueness of the mild solution. Furthermore, we gave an exact characterization for the time interval of its local existence in terms of the Coriolis force. It follows from our characterization that the existence time T of the solution can be taken arbitrarily large provided the speed of rotation is sufficiently fast.