Research plan

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1 Research on Harmonic Analysis

1.1 Research on the theory of weights

Research 1.3 of "Previous Research" have not be completed. The purpose is find the sharp lower bound of the ratio $\|f\|_{BMO(w)}/\|f\|_{BMO}$. The problem has several variations with other constants, for example, Sawyer's constant and Nazarov-Petermichl-Volberg's constant including heat semigroup.

Also, we challenge "two weight inequality". That is, I will be looking for a necessary and sufficiently condition on weights w and σ which ensures the boundedness of Calderón-Zygmung operators from $L^p(w)$ to $L^q(\sigma)$.

I try to give a characterization of weighted Hardy spaces, which were used in the previous research, in the sense of Littlewood-Paley.

1.2 Kakeya conjecture

I challenge to Kakeya conjecture which states that the Hausdorff dimension of a compact set in $\mathbb{R}^n$, whose Lebesgue measure is 0 and contains a unit line segment in every direction, is n. This conjecture is related to several problems, for example, Sogge's local smoothing estimates for some wave equations, restriction problems, Bochner-Riesz conjecture and the mapping properties of Kakeya maximal operator. In particular, I consider the second and last problems. Because the former is related to Strichartz's estimates for Schrödinger equations, I hope that this research give one approach to Research 2.2 below.

2 Research on PDE

2.1 Applications of weighted function spaces to fluid equations

In the previous research, our time decay orders of the energy of global solutions have not reached to one of Wiegner, but it is expected that making use of the real interpolation spaces of weighted Hardy spaces with respect to weight parameters allows global solutions to have the critical decay order due to Wiegner. I make sure of this expectation. Also, I'm going to consider the requirements for global solutions have faster decay order than the critical one of Wiegner. And, I make a search for applications of weighted function spaces to fluid equations, for example, vorticity equations and Euler equations.

2.2 Characterizations of function spaces defined by Schrödinger group and applications

In this research, I consider function spaces defined by Schrödinger group $S(t) = e^{it\Delta}$, for example, $\|f\|_{\dot{S}^{\alpha}_{p,q}} = \big\| |t|^{\alpha} \|S(t)f\|_{L^p(\mathbb{R}^n)} \big\|_{L^q(\mathbb{R})}$. The aim is to give characterizations of those function spaces without Schrödinger group $S(t)$. Those function spaces were applied by Cazenave-Weissler, Cazenave-Vega-Vilela, etc... for studies of non-linear Schrödinger equations. This research is motivated by several researches of Navier-Stokes equations with the Besov spaces $\dot{B}^{-\alpha}_{p,\infty}$, $(\alpha > 0)$, which are described by heat semigroup $e^{t\Delta}$. Because the conditions on indices in results of boundedness of PsDO or FIOp are similar to ones for Strichartz's estimates, I expect that characterizations of the function spaces are obtained by those operators. Also, to investigate those function spaces, I'm going to consider the inclusion relations between $\dot{S}^{\alpha}_{p,q}$ and Besov, Triebel-Lizorkin and modulation spaces. After these studies above, I will apply the function spaces to non-linear Schrödinger equations.