Research programs

A. **Relation between the center of mass and asymptotic behavior of solution to generalized Burgers equations**

In the previous works it is suggested that there is a deep relation between the center of mass and asymptotic behavior of solution to diffusion equations in the whole space. However most part of the works in the asymptotic behavior do not take into account the center of mass of the solution. For example, we refer to the case of generalized Burgers equations (M. Kato, Osaka Math. J., 44 (2007) 923–943).

The research aims to consider generalized Burgers equations in the whole space, and to study a relation between the center of mass and asymptotic behavior of solution.

B.**A parabolic-elliptic Keller-Segel system in $\mathbb{R}^2$**

We consider the following Keller–Segel system:

$$\text{(DD)} \qquad \begin{cases} \partial_t u = \Delta u - \nabla \cdot (u \nabla v), \quad 0 = \Delta v + u, & x \in \mathbb{R}^2, t > 0, \\ u(x,0) = u_0(x) \geq 0, & x \in \mathbb{R}^2. \end{cases}$$

It is well-known that the global existence and large-time behavior of nonnegative solutions to (DD) heavily depend on $\int_{\mathbb{R}^2} u_0(x)\,dx$. Indeed, for the case $\int_{\mathbb{R}^2} u_0(x)\,dx > 8\pi$, the nonnegative solution may blow up in finite time. On the other hand, for the case $\int_{\mathbb{R}^2} u_0(x)\,dx < 8\pi$, the nonnegative solution is globally defined in time, and it converges to a radially symmetric self-similar solution as $t \to \infty$. Also, in the case $\int_{\mathbb{R}^2} u_0(x)\,dx = 8\pi$, as for some choices of u_0 the nonnegative solution can grow-up, while for other choices of u_0 the solution approximates a steady-state as $t \to \infty$.

The research aims to study the following problems: in the case $\int_{\mathbb{R}^2} u_0(x)\,dx < 8\pi$,

(1) to improve the rate of convergence to a radially symmetric self-similar solution by taking into account the center of mass of the solution.

Furthermore, in the case $\int_{\mathbb{R}^2} u_0(x)\,dx = 8\pi$,

(2) to derive the rate of convergence to a steady-state.
(3) to construct the nonnegative solution to (DD) which oscillates.

C. **A parabolic-elliptic Keller-Segel system with special sensitivity functions**

We consider the following Keller-Segel system:

$$\text{(GDD)} \qquad \partial_t u = \Delta u - \nabla \cdot (u \nabla \psi(v)), \quad 0 = \Delta v - v + u, \quad x \in \Omega, t > 0,$$

where $\Omega \subset \mathbb{R}^2$ is either a bounded domain with smooth boundary $\partial\Omega$ or the whole space, and $\psi(v)$, which is called a sensitivity function, is either $\psi(v) = v^p$ $(p > 0)$ or $\psi(v) = \log v$.

When Ω is the open ball at the origin in $\mathbb{R}^2$, and a radially symmetric L^1-function $u(x,0)$ is nonnegative, Professors Toshitaka Nagai and Takasi Senba studied whether the blow-up of nonnegative solutions to (GDD) can occur or not under homogeneous Neumann boundary conditions (T. Nagai, T. Senba, Adv. Math. Sci. Appl. 8 (1998) 145–156). However there are little works analyzed (GDD) without assuming that $u(x,0)$ is radially symmetric as far as my knowledge.

The research aims to treat the Cauchy problem of (GDD) in $\mathbb{R}^2$ without assuming that $u(x,0)$ is radially symmetric, and to study the following problems:

(1) the global existence and blow-up of nonnegative solutions.
(2) the asymptotic behavior of nonnegative global solutions.

Moreover it is clear how sensitivity functions $\psi(v)$ influence the structure of solutions to (GDD).