Research Statement

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It is usually hard to obtain an explicit formula of the Bergman kernel of a given complex domain. Therefore it is fundamental and important to find a domain with explicit Bergman kernel. Actually, up to now many mathematicians work in that direction.

My current research interest is to find new domains with explicit Bergman kernels. I am also interested in the applications of our explicit formulas to various problems. Especially I am interested in the Lu Qi-Keng problem.

In my doctoral dissertation, I proved that the Bergman kernel of the Hartogs domain is expressed explicitly by the polylogarithm functions if its base domain satisfies a certain condition. I also found some concrete examples which satisfy the condition. For example, the domain $D_{n,m} = \{(z, \zeta) \in \mathbb{C}^n \times \mathbb{C}^m; \|\zeta\|^2 < e^{-\mu||z||^2}\}(\mu > 0)$ is a such domain.

As an application of my formula, I investigated the Lu Qi-Keng problem of the domain $D_{n,m}$. A domain in $\mathbb{C}^n$ is called a Lu Qi-Keng domain if its Bergman kernel function has no zeros. The Lu Qi-Keng problem asks whether or not a given domain is a Lu Qi-Keng domain.

I proved the following result for the domain $D_{n,m}$.

Theorem. *For any fixed $n \in \mathbb{N}$, there exists a unique number $m_0(n) \in \mathbb{N}$ such that $D_{n,m}$ is a Lu Qi-Keng domain if and only if $m \geq m_0(n)$.*

Another concrete example is the Cartan-Hartogs domain, which is defined as follows: $D_N = \{(z, \zeta) \in \Omega \times \mathbb{C}^m; ||\zeta||^2 < N(z, z)^\mu\}$, where Ω is a irreducible bounded symmetric domain and N its generic norm. An explicit formula of the Bergman kernel of this domain was firstly given by W. Yin. However the polylogarithm did not appear in his formula. Thus my formula is another expression of the Bergman kernel of the Cartan-Hartogs domain. L. Zhang and W. Yin (J. Math. Anal. Appl., 2009) obtained a result on the Lu Qi-Keng problem for the domain:

$$D_s := \{(z, \zeta) \in \mathbb{C}^n \times \mathbb{C}^m; \|z\|^2 + \|\zeta\|^{2s} < 1\},$$

which is a special case of the Cartan-Hartogs domain. I also succeeded in generalizing their result for the Cartan-Hartogs domain.