Abstract of the results of my research

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The aim of representation theory of algebras is to investigate the structure of their module categories. So far I mainly have been studying the derived categories, the equivalences between them and some invariants under derived equivalences. They are useful tools in representation theory of algebras. For example, the important information of algebras such as the Grothendieck group and the finiteness of the global dimension is invariant under derived equivalences. Recently, it turns out that they play important roles in Lie theory and noncommutative algebraic geometry.

The most important of various homological invariants in representation theory is the global dimension. The global dimension can be arbitrarily large. It thus reflects a minute property of the module categories.

Auslander experimentally proposed representation dimension, based on an idea that the representation theoretic properties such as the representation-finiteness must be controlled by the homological invariants such as the global dimension. This concept was not handled easily. The representation dimension is, however, important because it has provided many interesting issues in representation theory of algebras and has become one of the origins of cluster tilting theory proposed recently.

An important concept introduced recently in connection with the representation dimension is Rouquier's dimension of triangulated category. This is an attempt to formulate in category theory the geometric 'dimension' as the global dimension is. The derived dimension and the stable dimension of algebras are closely related to the global dimension and the representation dimension (Rouquier) so that they are especially interesting in representation theory of algebras. Note that the derived dimension and the stable dimension are invariant under derived equivalences (Rickard).

I hence considered that the representation theoretic properties of algebras could be controlled by the derived dimension and the stable dimension. First, I would investigate the representation theoretic properties of selfinjective algebras having low stable dimension. By definition, if a selfinjective algebra is representation-finite, then it has stable dimension 0. Then a natural question arises as to whether the converse should also hold. I gave an affirmative answer to this [2]. Although this was expected to hold by some experts, it had not been proved before. As an application, I exhibited in [4] (or [3]) that algebras having derived dimension 0 are closely related to selfinjective ones having stable dimension 0, improving Chen-Ye-Zhang's result. By using the same tool as the improvement, I also verified that any standard selfinjective algebra of polynomial growth has stable dimension at most 1 (see [3]).

By the way, I would like to decide the derived dimensions of wild canonical algebras to give an example of a selfinjective algebra having stable dimension 2 (Question A). Also, I would like to investigate the representation theoretic properties of selfinjective algebras having low derived dimension (Question B). Therefore, I would investigate the derived dimensions of algebras, whose class contains selfinjective algebras and canonical algebras. These algebras are called Iwanaga-Gorenstein. Namely, their selfinjective dimensions are finite. We showed in [5] that for an Iwanaga-Gorenstein algebra having selfinjective dimension d, it has derived dimension over the category of Cohen-Macaulay (=CM) modules at most $d' = \max\{d, 1\}$ (This is joint work with Aihara, Araya, Iyama and Takahashi). In particular, if an Iwanaga-Gorenstein algebra is of finite CM representation type, then it has derived dimension at most d'. One can recover Rouquier's result that the global dimension gives an upper bound of the derived dimension. After this, I will consider the Question A and B above by using this.