

# Research results

I am interested in topology and geometry of spaces with symmetry. In particular, my research interests focus on Hamiltonian torus actions on symplectic manifolds/orbifolds and their relations with combinatorics. Below, I summarize my research results so far.

## A generalization of convexity theorem of moment maps

For a Hamiltonian torus actions on a compact symplectic manifold, Atiyah and Guillemin-Sternberg showed, in 1982, that the image of the moment map is a convex polytope, the *moment polytope*. It is known that the combinatorial information of the moment polytope reflects the equivariant topology of the symplectic manifold. Kirwan generalized their theorem to the case for Hamiltonian actions of compact connected Lie groups. I gave a different generalization of the original convexity theorem with three Hamiltonian torus actions ([1]). The idea of three torus actions was motivated by the research of integrable structures of the Toda lattice by Agrotis-Damianou-Sophocleous. A generalization of the Delzant's classification of the symplectic toric manifolds in terms of moment polytopes is a future problem.

## Schubert calculus for weighted Grassmannians

This research is a joint work with Tomoo Matsumura (KAIST). Schubert calculus has its origin in counting the number of lines with certain conditions in a complex affine space. In the language of the modern mathematics, it is a problem of computations of the *structure constants* of the cohomology of the complex Grassmannian with respect to the Schubert classes. Topology, geometry, representation theory and combinatorics meets there via these structure constants. In [2], we developed Schubert calculus for the *weighted Grassmannian orbifold*. In particular, we introduced a natural definition of Schubert classes of the cohomology of the weighted Grassmannian, and computed the structure constants with respect to them. To do that, we in fact formulate the same problem in equivariant cohomology of the natural torus action on the weighted Grassmannian, and calculated the equivariant structure constants. The formula of our structure constants are described by the *equivariant* structure constants of the non-weighted Grassmannian.

It is known that Schubert classes for Grassmannian are represented universally by Schur polynomials. We introduced a generalization of Schur polynomials with weights where they do the same role for the weighted Schubert classes ([3]). So we expect that understanding their properties will lead us to a deeper understanding of Schubert calculus for weighted Grassmannians. For example, seeking the role of these generalizations in the representation theory of general linear groups is an interesting problem.