Research plans for 2013

(1) I observed that from a certain function on a vector space one can construct a linear representation of a Lie group admitting an open orbit. Since this observation is about a particular case, I will consider that construction in a more general setting. Firstly I will investigate whether a map $F : G \to GL(V)$, where G is a constructed Lie group, is a homomorphism or not. Secondly since this construction is depending on several parameters, we may obtain several representations F_λ of Lie groups G_λ. Thus one problem is constructed Lie groups G_λ are isomorphic or not. Moreover I will investigate the totality of representations obtained by that way.

(2) Let $\mathfrak{g} = \sum_{k=-\nu}^{\nu} \mathfrak{g}_k$ be a real simple graded Lie algebra, and G_0 be the group of graded-preserving automorphism of $\mathfrak{g}$. The orbit decomposition of $\mathfrak{g}_{-1}$ by the group G_0 in the case of $\nu = 1$ has been done by S.Kaneyuki.

I will investigate the general case $\nu \geq 2$ and especially determine which SGLA of the second kind admits an open orbit. About the classical simple Lie algebras, all gradations of the second kind are determined in Kaneyuki and Asano's paper, so it can be an important reference.

(3) This is a joint work with Dr. S.Kuroki. We try to share a problem and recognize the difficulty of it. Probably we will find a problem among the following topics: Determination of the cohomology rings of manifolds obtained by successive castling transformations. The orbit decomposition of products of Grassmannian manifolds by a projective linear group. To consider projective structures on the projectivation of vector bundles over manifolds equipped with a flat projective structure. Characterization of small covers admitting a flat projective structurer by polytopes. To consider compactifications of open orbits given by linear representations of Lie groups and GKM graphs which may be defined over compactifications.