

# Research plans

## Higher-dimensional Kaluza-Klein black holes with a twisted extra dimension

I would like to generalize a family of five-dimensional squashed Kaluza-Klein black hole solutions [1, 2, 4, 8, 9, 10, 11]. For example, I have constructed extremal charged solutions in odd dimensions, where the mass of the black hole is equal to its charge [14]. There exists a null hypersurface where an expansion for an outgoing null geodesic congruence vanishes, then these spacetimes look like black holes. The metrics admit  $C^0$  extension across the horizon, but some components of Riemann curvature diverge there if the dimension is higher than five. The singularity is relatively mild so that an observer along a free-fall geodesic can traverse the horizon.

When the size of a compact extra dimension becomes constant, I have investigated five-dimensional rotating Kaluza-Klein vacuum multi-black hole solutions [15]. Each black hole has a smooth horizon with the topology of the lens space. The mass of each black hole is quantized by the size of the compactified extra dimension. I have generalized such solutions to charged rotating Kaluza-Klein multi-black hole solutions in the five-dimensional pure Einstein-Maxwell theory [16]. Then I construct five-dimensional rotating Kaluza-Klein multi-black holes on the Taub-bolt space [17]. I also construct extremal charged black hole solutions in five-dimensional dynamical Kaluza-Klein universes [18]. Further I extend above discussion about analyticity of horizons of Kaluza-Klein multi-black holes in five dimensions to that in higher dimensions [19].

## Verification of extra dimension by squashed Kaluza-Klein black holes

Higher-dimensional black holes gather much attention not only as key points of unified theory but also in various fields of theoretical physics. For example, five-dimensional squashed Kaluza-Klein black hole solutions behave as fully five-dimensional black holes in the vicinity of the squashed  $S^3$  horizon, while they asymptote to four-dimensional flat spacetimes with a twisted  $S^1$  as a compactified extra dimension. Then we can regard these squashed Kaluza-Klein black hole solutions as models of realistic higher-dimensional black holes.

A five-dimensional squashed Kaluza-Klein black hole spacetime admits stable circular orbits similar to the four-dimensional black hole spacetimes. I focus on the geodetic precession effect, the classical tests (light deflection, time delay, and perihelion precession), innermost stable circular orbits, and black hole shadows. As a first step, I have considered the parallel transportation of a spin vector along a circular geodesic in the squashed Kaluza-Klein black hole spacetime and derived the geodetic precession angle with the higher-dimensional correction [12]. Then I discuss remained physical phenomena with higher-dimensional corrections. If precise experiments of these phenomena agree with the expected values of general relativity, it requires a rigorous upper limit of the size of the extra dimension, or it excludes the squashed Kaluza-Klein metric for describing the geometry around astronomical objects.