Research outline

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We may regard a maximally non-integrable hyperplane distribution on a (2n + 1)-manifold M^{2n+1} as a Γ -structure by the pseudogroup of local contactomorphisms of the 1-jet space $J^1(n, 1)$ for a function of n variables, and call it a contact structure. Also we may regard a pair $([\alpha], [\omega])$ of conformal classes of 1- and 2-forms α, ω with $\alpha \wedge \omega^n > 0$ as a G-structure on M^{2n+1} by the group of linear contactomorphisms of $J^1(n, 1)$, and call it an a(lmost)-c(ontact) structure. $([\alpha], [d\alpha])$ is a-c if $\alpha = 0$ is contact. We can associate an a-c structure to a given codimension one leafwise symplectic foliation (regular Poisson structure). Including them, I generalized the notion of confoliation due to Eliashberg and Thurston into a class of higher dimensional a-c structures (different from the Altshular-Wu generalization). Recently, motivated by an effort of Verjovsky, Mitsumatsu constructed a leafwise symplectic foliation on S^5 . I [11] constructed a path of confoliations connecting the standard contact structure on S^5 with Mitsumatsu's structure.

Seifert surfaces in $J^1(1,1) \approx S^3 \setminus \{*\}$ satisfies Bennequin's inequality, and surfaces in a contact 3-manifold are smoothly approximated by 'convex' ones. I [10] constructed a Seifert hypersurface in $J^1(2,1) \approx S^5 \setminus \{*\}$ which violates the inequality and is not approximated by a 'convex' hypersurface. Lutz modified the contact structure of $J^1(1,1)$ into exotic one. Using geometry of Brieskorn 3-manifolds, I [9] generalized the Lutz modification into $J^2(2,1)$. I obtained a 'convex' Seifert hypersurface which violates the inequality and obstracts symplectic fillability.

In [4] and [13] I constructed a certain immersion of a given contact M^3 to $J^1(2, 1)$ by using approximately holomorphic geometry. This result has been generalized by Martínez Torres. In [8] I smoothly isotoped the standard S^3 in $J^1(2, 1) \approx$ $S^5 \setminus \{*\}$ so that the restricted contact structure converges to the Reeb foliation (by Legendrian submanifolds of S^5); and then becomes to an exotic contact structure. I explained the non-analyticity of Reeb foliation by using toric geometry on S^5 .

Thurston and Winkelnkemper constructed a contact structure on a given openbook 3-manifold. I [3] showed that it comes from a symplectic filling if the monodromy is 'positive(right-handed)'. Loi and Piergallini showed that a 3-manifold is diffeomorphic to the boundary of a Stein domain iff it admits a 'positive' openbook. These results are later unified in Giroux's one-to-one correspondance between contact structures and stable positive stabilizations of open-books. I also showed that any contact structure on M^3 can be deformed into a spinnable foliation. This implies that the relative Thurston inequality holds for many foliations with Reeb components in contrast to the Eliashberg-Thurston theory. With collaborators, I obtained relevant results: See [7] for homological overtwistedness, [6] for Dehn fillings, and [5] for a generalization of Bennequin's isotopy lemma.

I also have a collaboration [1] with Fukui on (in)stability of certain foliations.