RESEARCH TO DATE

KAZUKI MORIMOTO

0.1. **Tensor product L-functions for quaternion unitary groups and GL_2.** In [5], I gave an integral representation for tensor product L-functions for GL(2) and a quaternion unitary group G_D. Here we note that $G_D \simeq \mathrm{GSp}(4)$ when $D = M_2(\mathbb{Q})$. Then using this integral representation, I proved the following theorem.

Theorem 1 (Morimoto [5]). *Suppose that $D \otimes_{\mathbb{Q}} \mathbb{R} \simeq M_2(\mathbb{R})$. Let (Π, V_Π) be an irreducible unitary cuspidal automorphic representation of $G_D(\mathbb{A}_{\mathbb{Q}})$ such that Π_∞ is the discrete series representation with the Harish-Chandra parameter $(\ell-1, \ell-2)$ for $\ell > 6$. Let (σ, V_σ) be an irreducible cuspidal automorphic representation of $\mathrm{GL}(2, \mathbb{A}_{\mathbb{Q}})$ such that $\sigma_\infty \otimes |\cdot|^{-\frac{u}{2}}$ is the holomorphic discrete series of weight ℓ. Let $G(\nu_0)$ be the Gauss sum for $(\omega_\sigma \cdot \omega_\Pi)^{-1} / |\omega_\sigma \cdot \omega_\Pi|^{-1}$. For $\Phi \in V_\Pi$ and $\Psi \in V_\sigma$ which are arithmetic over $\overline{\mathbb{Q}}$ in the sense of Harris [4] and each integer m such that $2 + \frac{u}{2} < m \leq \frac{\ell+u}{2} - 1$, we define*

$$A(m, \Pi, \sigma, \Phi, \Psi) := \frac{L(m, \widetilde{\Pi} \times \widetilde{\sigma})}{\left(\sqrt{-1}\right)^{\ell} \pi^{4m-2u} G(\nu_0)^2 \langle \Psi, \Psi \rangle \langle \Phi, \Phi \rangle} \tag{0.1}$$

where $\widetilde{\Pi}$ and $\widetilde{\sigma}$ denote the contragredients of Π and σ respectively.

Then $A(m, \Pi, \sigma, \Phi, \Psi) \in \overline{\mathbb{Q}}$ and $A(m, \Pi, \sigma, \Phi, \Psi)^\tau = A(m, \Pi^\tau, \sigma^\tau, \Phi^\tau, \Psi^\tau)$ for any $\tau \in \mathrm{Gal}\left(\overline{\mathbb{Q}}/\mathbb{Q}\right)$.

As a corollary of this theorem, I proved a relation between Petersson inner products on GSp_4 and G_D assuming the existence of Jacquet-Langlands correspondence of these groups, which is expected to be proved as a consequence of Arthur's book project.

0.2. **Tensor product L-functions for $\mathrm{SO}(V)$ and GL_2.** In a joint work with Furusawa [3], we considered special values of L-functions for $\mathrm{SO}(V) \times \mathrm{GL}_2$ where V is an orthogonal space over $\mathbb{Q}$ which is anisotropic over $\mathbb{R}$.

Theorem 2 (Furusawa-Morimoto [3]). *Let σ be an irreducible cuspidal automorphic representation of $\mathrm{SO}(V, \mathbb{A}_{\mathbb{Q}})$ such that the archimedean component σ_∞ is the trivial representation. Let f be a holomorphic newform of weight k, and τ the irreducible cuspidal automorphic representation of $\mathrm{GL}_2(\mathbb{A}_{\mathbb{Q}})$ associated to f. Suppose that $k > 2n$ where $n = \dim_{\mathbb{Q}} V$. Then there exists a finite set S of places of containing the real place such that the partial L-function $L^S(s, \sigma \otimes \tau)$ satisfies*

$$\frac{L^S\left(\frac{k-n+1}{2}, \sigma \otimes \tau\right)}{\pi^{[\frac{n}{2}]\cdot(2k-n)} \langle f, f \rangle^{[\frac{n}{2}]}} \in \overline{\mathbb{Q}}.$$

When $\dim_{\mathbb{Q}} V = 4$, by the accidental isomorphism, any irreducible automorphic representation of $\mathrm{SO}(V, \mathbb{A}_{\mathbb{Q}})$ may be regarded of the form $\sigma_1 \otimes \sigma_2$ where σ_i is an irreducible automorphic representation of $D^\times(\mathbb{A}_{\mathbb{Q}})$ such that $\omega_{\sigma_1}\omega_{\sigma_2} = 1$, where D is a definite quaternion algebra. Thus the above theorem yields a result on algebraicity of special values of Rankin triple L-functions for GL_2. This theorem falls within the "unbalanced" case, i.e. $k_1 > k_2 + k_3$, and conforms with the explication by Blasius [1] of Deligne's conjecture.

0.3. **Shalika periods on $\mathrm{GU}(2,2)$.** In a joint work with Furusawa [2], we gave an integral representation of twisted exterior square L-functions for irreducible cuspidal generic automorphic representation π of $\mathrm{GU}(2,2)$ and proved the following theorem.

Theorem 3 (Furusawa-Morimoto [2]). *For an irreducible cuspidal unitary automorphic representation π of $\mathrm{GU}(2,2)(\mathbb{A}_F)$ whose central character ω_π satisfies $\omega_\pi |_{\mathbb{A}_F^\times} = \xi^{-2}$, the following two conditions are equivalent:*

(1) *The Shalika period with respect to ξ does not vanish on the space of π.*
(2) *π is globally generic and the partial twisted exterior square L-function $L^S\left(s, \pi, \wedge_t^2 \otimes \xi\right)$ has a simple pole at $s = 1$.*

References

[1] D. Blasius: Critical values of certain tensor product L-functions. Appendix to T. Orloff, Special values and mixed weight triple products. *Invent. Math.* **90**, 181–188 (1987).
[2] M. Furusawa and K. Morimoto: Shalika periods on GU(2, 2). Accepted for publication in *Proc. Amer. Math. Soc.*
[3] M. Furusawa and K. Morimoto: On special values of certain L-functions. Submitted.
[4] M. Harris: Arithmetic vector bundles and automorphic forms on Shimura varieties. I. *Invent. Math.* **82**, 151–189 (1985).
[5] K. Morimoto: On L-functions for quaternion unitary groups of degree 2 and GL(2) (with an Appendix by M. Furusawa and A. Ichino). Accepted for publication in *Int. Math. Res. Not. IMRN.*