Plan of Research

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I calculate the bridge number and the braid number of a lens space $L(p,q)$. Let $[a_1, a_2, \cdots, a_n]$ be a continued fraction of $\frac{p}{q}$. I have the following results. If $n = 1$ then

$$g_{\text{bridge}}(L(p,q)) = g_{\text{braid}}(L(p,q)) = 3.$$

If $n = 2$ then

$$g_{\text{bridge}}(L(p,q)) = g_{\text{braid}}(L(p,q)) = 4.$$

It seem to $g_{\text{bridge}}(L(p,q)) = g_{\text{braid}}(L(p,q)) = n + 2$.

Next, we interest in the case of $n = 3$. I want to calculate $g_{\text{bridge}}(L(8,3))$ and $g_{\text{braid}}(L(8,3))$. I hope for $g_{\text{bridge}}(L(p,q)) = g_{\text{braid}}(L(p,q)) = 5$. We have $4 \leq g_{\text{bridge}}(L(8,3)) = g_{\text{braid}}(L(8,3)) \leq 5$. I want to prove $g_{\text{bridge}}(L(p,q)) = g_{\text{braid}}(L(p,q)) = 5$ by using the Rohlin invariant.

It is well known that every closed connected orientable 3-manifold M is obtained by the 0-surgery of S^3 along a link. We can show that every M is obtained by the 0-surgery of S^3 along a link L such that each component of link is unknot. The minimal of component numbers of L is a new invariant of M. I will consider this invariant.

I interest in the handlebody knot theory. A simple closed curve on the boundary of a handlebody which embedded in S^3 is a knot. In general, there exist infinite many simple closed curve on the boundary of a handlebody. By picking up the essential curve in these curves and using knot invariants, I want to classify handlebody knots.