results of research

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A knot is the image of an embedding of circle in the 3-sphere S^3, denoted by K. A link is the image of an embedding of circles $S^1 \cup S^1 \cup \cdots \cup S^1$ in the 3-sphere S^3. Let $L = K_1 \cup K_2 \cup \cdots \cup K_n$ be an n-component link in S^3, and $N(L)$ a tubular neighborhood of L, and $E(L)$ the exterior of L. Let $\chi(L, 0)$ be the 3-manifold obtained from $E(L)$ by attaching n solid tori $V_1, V_2, \ldots, V_n$ to $\partial E(L)$ such that the meridian of ∂V_i is mapped to the longitude of K_i $(i = 1, 2, \ldots, n)$. We call $\chi(L, 0)$ the *3-manifold obtained by the 0-surgery of* S^3 along L. It is well known that every closed connected orientable 3-manifold is obtained by the 0-surgery of S^3 along a link.

Let M be a closed connected orientable 3-manifold. For any M, there exist handlebodies H_1 and H_2 of same genus and a homeomorphism $f : \partial H_1 \to \partial H_2$ such that $M = H_1 \cup_f H_2$. We call the triple $(H_1, H_2; f)$ a *Heegaard splitting* of M and we call $f(\partial H_1) = \partial H_2$ the *Heegaard surface*. The *Heegaard genus* of M is the minimal genus of Heegaard surfaces, denoted by $g_{\mathrm{H}}(M)$.

Let bridge(L) (resp. braid(L)) be the bridge index (resp. the braid index) (cf. [5]). The *bridge genus* $g_{\mathrm{bridge}}(M)$ (resp. the *braid genus* $g_{\mathrm{braid}}(M)$) of M is the minimal number of bridge(L) (resp. braid(L)) for any L such that M is obtained by the 0-surgery of S^3 along L. The bridge genus and the braid genus are introduced by A.Kawauchi [6].

I show the following results and my paper *On Heegaard genus, bridge genus and braid genus for a 3-manifold* is published in Journal of Knot Theory and Its Ramifications.

$$g_{\mathrm{H}}(M) \leq g_{\mathrm{bridge}}(M) \leq g_{\mathrm{braid}}(M).$$

Next, I calculate the bridge genus and the braid genus for lens space $L(p, q)$ as follows. For every even integer n,

$$g_{\mathrm{bridge}}(L(n, 1)) = g_{\mathrm{braid}}(L(n, 1)) = 3.$$

For every odd integers n, m,

$$g_{\mathrm{bridge}}(L(nm - 1, m)) = g_{\mathrm{braid}}(L(nm - 1, m)) = 4.$$

For lens space $L(8, 3)$,

$$4 \leq g_{\mathrm{bridge}}(L(8, 3)) = g_{\mathrm{braid}}(L(8, 3)) \leq 5.$$