The plan of future study

We continue to study the spectral analysis of Hamiltonians (we denote it by H) for non-relativistic infinitely many boson systems and prove the asymptotic completeness. It is well known that the system we consider is described by non-linear Schrodinger equation (NLS) under the semi-classical approximation. In the theory of NLS, the scattering matrix can be also defined, although the existence is not trivial at all. We study the relation between the scattering matrix of the quantum system and that of classical system. Next we study superradiant scattering, which is the scattering phenomena between light and an ensemble of atoms (BEC), in view of mathematical physics. This research will be divided into the following three parts:

1. Asymptotic completeness for the Hamiltonians. Physically, under the scattering state, particles expect to be scattered constructing clusters. In mathematical view point, it can be proved by studying the asymptotic analysis of the Heisenberg operator of the distance between particles. First, we prove so-called large velocity estimates, which say roughly that the velocity of a particle is bounded by its energy. Next, we prove that each cluster moves according to the classical orbit (Phase space propagation estimate). Finally, we prove that under the eigenstates of H, the distance between particles will be asymptotically zero as time goes to infinity (Minimal velocity estimates). It is most difficult to prove the last propagation estimates. We overcome the difficulty by using the previous result (1-b) and Higher order estimates.

2. Semi-classical analysis. The system we consider can be regarded as the quantum version of the classical field, which is called de-Broglie field. In this classical field, the system is described by non-linear Schrodinger equation (NLS). *Formally*, NLS can be derived from the equation of quantum de-Broglie field using semi-classical analysis. As is well known, semi-classical analysis for quantum mechanics gave the deep insight into the various filed of analysis, such as micro local analysis and harmonic analysis. Hence it is interesting to develop the semi-classical analysis for quantum field theory. In this step, we study the quantum-classical correspondence for non-relativistic infinitely many boson systems. First we consider the Weyl operators generated by the solution of NLS. Physically, this operator can be regarded as the projection onto the space of coherent vectors. Next we prove that, in the semi-classical limit, the product of Weyl operator and time evolution operator is Wave operator of NLS. Adding this with the results obtained in the first step, we prove that the scattering matrix of the quantum theory corresponds with that of NLS.

3. Superradiant scattering from a BEC. A Bose-Einstein condensate (BEC) is a state of matter of a dilute gas of bosons cooled to temperatures very near zero. Under such conditions, a large friction of the bosons occupy the lowest quantum state, at which point quantum effects become apparent on a macroscopic scale. In 1995, the first gaseous condensate was produced. After that , BEC has been studied by many mathematicians and theoretical physicists. In this research, we study the scattering phenomena which is so-called superradiant scattering. It causes by the interactions of lights with an ensemble of atoms. In mathematical point of view, we construct the Hamiltonians which describe a quantum de-Broglie field interacting with photons. Next, we study the spectral analysis for such Hamiltonians.

In order to carry out the plan, heuristic arguments with respect to physics will be required. Hence I will join the conference of physics and mathematics to be held domestic and abroad