Research results

Kiyoki Tanaka

I have been studied the Bergman space theory. The Bergman spaces $L^p_a(\Omega)$ (resp. the harmonic Bergman spaces $b^p(\Omega)$) over some domain Ω are defined spaces of all holomorphic functions (resp. harmonic functions) on Ω which are p-th integrable functions. When $p = 2$, it is known that the Bergman space (resp. harmonic Bergman space) is a reproducing kernel Hilbert space, its reproducing kernel is called Bergman kernel (resp. harmonic Bergman kernel). In particular, when a domain is the unit ball $\mathbb{B}$, then it is known the explicit form of the Bergman kernel. By using the explicit form of the Bergman kernel of the unit ball $\mathbb{B}$, C. A. Berger, L. A. Coburn and K. H. Zhu studied Toeplitz operators, Hankel operators etc. on the Bergman spaces over the unit ball. On the other hand, C. Fefferman gave the boundary behaviors of the Bergman kernels of the pseudo-convex domains with smooth boundaries. It is an interesting problem what is the best condition of domains for analyzing the behavior of the Bergman kernel or investigating the operators on Bergman space. I studied the harmonic Bergman spaces over bounded smooth domains. The reproducing kernel of general domain does not have the explicit form. Recently, H. Kang and H. Koo obtained the estimates for the harmonic Bergman kernel of a bounded smooth domain. I obtained the following results based on the above results.

1. Atomic decomposition theorem

R. R. Coifman and R. Rochberg gave the series representation consisted of the reproducing kernel for harmonic Bergman functions on the unit ball. The above representation is called atomic decomposition. I obtained the atomic decomposition of harmonic Bergman functions on bounded smooth domains for $p > 1$ ([1]).

2. Modified harmonic Bergman kernel

I considered the series representation for functions in b^1. By using the modified harmonic Bergman kernel, which is constructed by B. R. Choe, H. Koo and H. Yi, we obtain a series representation for functions in b^1. I also considered the harmonic Bloch space, which is identified with the dual of b^1 and gave a similar series representation for harmonic Bloch functions ([2]). As an application, I considered Toeplitz operator. B. R. Choe, Y. Lee and K. Na gave characterizations for corresponding Toeplitz operators to be bounded, compact and in the Schatten classes. By using series representation, I extended their results.