

Previous Research

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1 Research on Harmonic Analysis

1.1 Boundedness of PsDO of class $S_{0,0}^m$ on weighted spaces

This study was motivated by L^2 -boundedness result due to Calderón-Vaillancourt and $H^p - L^p$ boundedness results due to Miyachi. The aim was to find necessary conditions on m which ensures the $H\dot{K}_{p,q}^\alpha - \dot{K}_{p,q}^\alpha$ boundedness of PsDO of class $S_{0,0}^m$, where $\dot{K}_{p,q}^\alpha$ are Herz spaces and $H\dot{K}_{p,q}^\alpha$ denotes the Hardy type variants $\dot{K}_{p,q}^\alpha$. Our main tools were atomic decompositions for $H\dot{K}_{p,q}^\alpha$ and complex interpolation theory for the bilinear operator $S_{0,0}^m \times H\dot{K}_{p,q}^\alpha \rightarrow \dot{K}_{p,q}^\alpha$; $(\sigma, f) \mapsto \sigma(X, D)f$ with Miyachi's results.

1.2 Bilinear estimates by the sharp maximal operator

Estimates for the bilinear forms $f\nabla^k g$ on Herz spaces and Morry spaces have given. Firstly, Kozono-Taniuchi established several estimates for the bilinear forms $f\nabla g$ on Lebesgue spaces. Here, we followed the argument of Miyachi which uses the sharp maximal operator.

1.3 A_∞ constants between BMO and weighted BMO

For any Calderón-Zygmund operators T , to give a sharp estimate of $\|T\|_{L^\infty \rightarrow BMO(w)}$, where $BMO(w)$ denotes weighted BMO with $w \in A_\infty$, we investigated the control of the ratio $\|f\|_{BMO(w)}/\|f\|_{BMO}$ from above and below in terms of A_∞ constant of w . The sharp inequality from above was showed by Hytönen-Pérez. We gave two other proofs of the upper bound of ratio and a lower bound.

2 Research on the incompressible Navier-Stokes equations

2.1 Well-posedness on weak Herz spaces

To consider the well-posedness on function spaces, which measure the decay properties of functions, for the equations, we used weak Herz spaces $W\dot{K}_{p,q}^\alpha$. Our largest spaces on which we can construct global solutions with small data includes a function having a singular point on each dyadic annulus. We have to remark that our largest space is included in BMO^{-1} which was dealt by Koch-Tataru. Uniqueness on those spaces were studied with Meyer's method.

2.2 Time decay estimates with weighted Hardy spaces

In previous research, the function spaces $L^p(w)$ with $w(x) = |x|^{\alpha p}$ where $-n/p \leq \alpha \leq n(1 - 1/p)$ were dealt. The condition on α implies $w \in A_p$. This restrictions on weights are natural from a point of view of the weight theory in harmonic analysis. To get rid of the restriction, the weighted Hardy spaces $H^p(w)$ were applied. As a result, if the initial data belongs to L^n and the suitable $H^p(w)$ with small L^n norm, then the decay order of the energy $\|u(t)\|_{L^2}$ of global solutions with the initial data can be close to a critical one due to Wiegner as possible. To control the non-linear term $(u \cdot \nabla)u$, weighted version of div-curl lemma by Coifman-Lions-Meyer-Semmes was applied. The div-curl lemma was proved from an estimate for maximal function of the bilinear form and an approach due to Auscher-Russ-Tchamitchian.