Plan

On exact solutions of Lieb-Liniger model and its extensions

Lieb-Liniger model is a many body quantum mechanical model proposed in 1963 [1]. This is simple model where N bosonic particles on the circle interact each other via δ-function potential and exactly solved by Lieb and Liniger themselves [1]. There are a lot of extensions, boson to anyon [2], circle to line [3], etc.. These extensions are also exactly solvable models. It is known that Lieb-Liniger model is equivalent to second quantized Gross-Pitaevskii equation without potential term and many researches have been made in mathematical physics [4]. For example, J. Sato et. al. obtain superposition of state in Lieb-Liniger model that correspond to soliton of Gross-Pitaevskii equation [5].

Since I achieved the results on the study of the Lieb-Liniger model in 2012, I am planning continuation of the further research also in 2013.

(1) On holonomy of Lieb-Liniger model's extension and its interpretations

δ-function potential is only a mere example of point-like interactions [6]. We can consider modification of a Lieb-Liniger model with point-like interactions other than δ-function potentials. Moreover, a lot of solvable extension of Lieb-Liniger model is known [2-3]. I plan to specify the nature of holonomy of such Lieb-Liniger model's extensions and clarify the relation to geometric properties of the parameter space of the models.

(2) Extensions of Lieb-Liniger model and Gross-Pitaevskii equation

There must be a second quantized field theory that is equivalent to an extended Lieb-Liniger model respectively, as Gross-Pitaevskii equation is equivalent to Lieb-Liniger model. I plan to find these second quantized field theories.

(3) Approximate energy in a weak coupling regime

We have already computed the approximate energy eigenvalues except some states in a weak attractive coupling regime [7]. I plan to obtain approximate energy eigenvalues for all state which is justified in a weak coupling regime. I also plan to obtain approximate energy eigenvalues of Lieb-Liniger model's extension.

Reference

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