Research statement

Schubert calculus for Springer varieties

Springer varieties are defined as subvarieties in the flag variety of general Lie type, and they are fundamental spaces for geometric representations for Weyl groups. The ring structure of the cohomology of the Springer variety is computed by Toshiyuki Tanisaki in Osaka City University. We aim to understand its ring structure in terms of restrictions of Schubert classes from the cohomology of the ambient flag variety. Since the relations in the cohomology ring of the Springer variety is well understood, we first study which Schubert classes can be chosen to form an additive basis of the cohomology of the Springer variety. After that, we will compute the expansion coefficients for a product of two classes from this basis, especially for the cases that one of those two are of degree 2 whose expansion coefficients are well known for the ambient flag variety. This research is based on the collaboration with Tatsuya Horiguchi in Osaka City University.

Chen-Ruan cohomology of weighted Grassmannians

As joint works with Tomoo Matsumura (KAIST), we computed the structure constants for the rational cohohomology of the weighted Grassmannian with respect to Schubert classes, and a presentation of the rational cohomology ring as a quotient of a polynomial ring whose variables are Chern classes of the tautological orbi-bundle will be appeared in the revised version of [3]. Since the weighted Grassmannian is an orbifold, it is interesting to study the Chen-Ruan cohomology ring as the next problem. Giving a presentation of the Chen-Ruan cohomology ring and its relation with Schubert calculus are also an interesting problem. The ultimate goal of this project would be the understanding of the quantum orbifold cohomology of weighted Grassmannians and its relation with Schubert calculus.

Toric manifolds associated withroot systems

Let G be a semisimple algebraic group and B its Borel subgroup. Studying the subvarieties of the flag variety G/B, called *Hessenberg varieties*, is recently an active research area. These subvarieties are studied from topology, geometry and representation theory. Among these, there is a distinct subvariety, the toric manifold associated withthe Weyl chambers of the root system of G. The intersections with Schubert cells of the flag variety provide a cell decomposition of the toric manifold, and hence give an additive basis of the cohomology. Since the intersection theory on the toric manifold computes these structure constants, it is natural to ask for a combinatorial formula of the structure constants with respect to these basis in terms of the root system.

Since the fan consisted of the Weyl chamber is a normal fan of the moment polytope of the flag variety G/B which is Delzant, we can also consider the moment-angle manifold of this polytope. It is also natural to ask whether we have a simple combinatorial expression for Euler numbers, Betti numbers, bigraded Betti numbers of our moment-angle manifold in terms of the root system. Also, since the Weyl group acts on our moment-angle manifold as above, it would be interesting to study the representation induced on their cohomology, as Stembridge studied the Weyl group representation on the cohomology of the above toric manifold.