Result

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The study of the alternation number

The **alternation number** of a knot is one of invariants which measures how the knot is complicated. Two properties of this invariant are the following:

- It is define by an elementary and natural way in terms of knot theory.
- It was not sufficiently studied.

In the paper [8], I gave a tool to study the alternation number of a knot using the Khovanov homology theory introduced in 1999. Concretely speaking, I gave a lower bound for the alternation number of a knot by using the Rasmussen invariant (which is derived from the Khovanov homology). I also affirmatively solved the conjecture in the book [A],

"The almost alternating torus knots are only type (3,4) or (3,5)"

by using this lower bound.

Reference [A] C. Adams, *The knot book*, American Mathematical Society, 2004.

The study of handle decompositions of a 4-manifold

A **handle decomposition** is a fundamental tool in the study of smooth manifolds. In the paper [1], I proved a theorem on the "diversity" or "flexibility" of handle decompositions of a 4-manifold. Concretely speaking, I constructed infinitely many 4-manifolds $\{X_i\}_{i\in\mathbb{Z}}$ with the following properties:

- X_i has a handle decomposition which consists of 0-handle and 2-handle h_i^2.
- For any integers $i \neq j$, h_i^2 and h_j^2 are mutually distinct[1]。
- For any integers $i \neq j$, X_i and X_j are diffeomorphic.

In the construction, we used a local move on knots, called the **annulus twist**, which was studied in the 3-manifold theory. I proved that the 4-manifold represented by the new handle decomposition obtained from the original one by using the annulus twist is diffeomorphic to the original 4-manifold.

The construction of potential counterexamples to the slice-ribbon conjecture

In the preprint [1], I constructed potential counterexamples to the slice-ribbon conjecture, which is one of the most important problems in knot theory. A property of this construction is that I used

the annulus twist (which was studied in 3-manifold theory).

In the proof, we used

the theory of 4-dimensional handle decompositions[2].

I also found that the annulus twist is related to the log transformation (which is a fundamental operation in the 4-manifold theory).

[1]It means that the attaching spheres (that is, knots) are mutually distinct

[2]Many people call it **Kirby calculus**.