Plan of study

1. Classification of homogeneous Lagrangian submanifolds in Kähler manifolds and the H-stablity.

   It is an important problem in symplectic geometry and differential geometry to classify homogeneous Lagrangian submanifolds in Kähler manifolds. For instance, any compact homogeneous Lagrangian submanifold is H-minimal, and the Hamiltonian stability is investigated by the harmonic analysis. Recently, Bedulli-Gori classified all compact homogeneous Lagrangian submanifolds in $\mathbb{C}P^n$, and Ma-Ohnita done in $Q_n(\mathbb{C})$ by using the moment maps. Moreover, Ma-Ohnita decided the H-stability of these submanifolds in $Q_n(\mathbb{C})$. When the case of $\mathbb{C}P^n$, Oh and Ohnita posed the problem: "Is any embedded compact minimal Lagrangian submanifold in $\mathbb{C}P^n$ H-stable?" To consider this problem, we want to investigate the H-stability of the homogeneous Lagrangian submanifolds in $\mathbb{C}P^n$.

   It is known that a homogeneous minimal submanifold in $\mathbb{R}^n$ is totally geodesic, namely, it is an affine plane [Di Scara 2002]. Moreover, any homogeneous submanifold $\mathbb{R}^n$ with parallel mean curvature vector is essentially contained in a sphere [Olmos 1994]. To classify the homogeneous H-minimal Lagrangian submanifolds in $\mathbb{C}^n$, we want to extend Olmos's result to H-minimal Lagrangians. Note that we have already constructed an H-minimal Lagrangian submanifold in $\mathbb{C}^n$ with cohomogeneity which is grater or equal to 1 as a normal bundle of a C-space.

   We want to consider the H-minimality of a singular orbit of the adjoint orbit of a compact semi-simple Lie group (namely, a generalized flag manifold). We also consider the H-stability and the Maslov class of normal bundles of C-spaces.

2. Classification of tight Lagrangian submanifolds

   A Lagrangian submanifold L in a Kähler manifold M is called (globally) tight if it satisfies $\#L \cap gL = SB(L, \mathbb{Z}_2)$ for any isometry g of M such that L transversally intersects with gL, where $SB(L, \mathbb{Z}_2)$ is the sum of Betti number of L. Y. G. Oh proved that the closed, embedded, (globally) tight Lagrangian submanifold in $\mathbb{C}P^n$ is only the totally geodesic $\mathbb{R}P^n$, and posed the classification problem of tight Lagrangian submanioflds in compact Hermitian symmetric spaces. For example, Tanaka-Tasaki proved that real forms in a compact Hermitian symmetric space are globally tight. On the other hand, due to the results of Gorodski-Podesta, we know that the tightness of Lagrangian submanifold is closely related to the notion of tight or taut immersions into Euclidean space. By using results in the theory of tight immersions, we want to classify the tight Lagrangian submanifolds in a specific Kähler manifold.