Results

1. Hamiltonian minimality of normal bundles over the isoparametric submanifolds [4]

In 90's, Y.-G. Oh introduced the notion of *Hamiltonian-minimal* (*H-minimal*) Lagrangian submanifolds in Kähler manifolds. Such a submanifold is a critical point of the volume functional under the Hamiltonian deformation. This is an extension of the notion of *minimal* submanifold, and has been studied by many authors. An H-minimal Lagrangian submanifold is called *Hamiltonian-stable* (*H-stable*) if the second variation is non-negative for any Hamiltonian deformation. Oh studied H-stablity of some examples of H-minimal Lagrangian submanifold in a specific Kähler manifold. For example, the real projective space $\mathbb{R}P^n$ in $\mathbb{C}P^n$ and the standard tori in $\mathbb{C}^n$ are H-stable. On the other hand, we know a few family of H-minimal Lagrangian submanifolds in $\mathbb{C}^n$. We proved that any normal bundle of a principal orbit N of the adjoint representation of a compact semi-simple Lie group G in the Lie algebra $\mathfrak{g}$ is an H-minimal Lagrangian submanifold in the tangent space $T\mathfrak{g} \simeq \mathbb{C}^n$. These orbits are called the complex flag manifolds or C-spaces. Moreover, we characterize C-spaces by this property in the class of full and irreducible isoparametric submanifolds in $\mathbb{R}^n$.

2. On the minimality of normal bundles and austere submanifolds [3]

Harvey-Lawson proved that the normal bundle of a submanifold N in $\mathbb{R}^n$ is special Lagrangian of some phase in $T\mathbb{R}^n \simeq \mathbb{C}^n$ if and only if N is an austere submanifold, namely, the set of principal curvatures of N is invariant by multiplication by -1 for any unit normal vector. Thus, an construction of austere submanifolds is an important problem in $\mathbb{R}^n$. However, we know little of properties and applications of austere submanifolds in general Riemannian manifolds. Let N be a submanifold in a Riemannian manifold M. We consider the normal bundle νN in the tangent bundle TM equipped with the Sasaki metric g_S, and investigate the relation of the minimality of νN and the austere condition of N. In (TM, g_S), there are natural almost complex structure, and the symplectic structure is identified with the standard one on the cotangent bundle T^*M. We prove that (i) When M is a simply connected symmetric space, νN is totally geodesic if and only if N is a reflective submanifold, (ii) When M is a real space form, νN is minimal if and only if N is an austere submanifold, (iii) When M is a non-flat complex space form, the normal bundles of totally geodesic submanifolds, complex submanifolds and austere Hopf hypersurfaces with constant principal curvatures are minimal. However, there exist an austere surface (namely, minimal surface) which dose not have minimal normal bundle.

3. Stability of Legendre submanifolds in Sasaki manifolds [1]

There is a notion of *Sasaki manifolds*, which is an odd-dimensional counterpart to Kähler manifolds. In Sasaki manifolds, we consider *Legendrian-minimal* (*L-minimal*) Legendrian submanifolds which correspond to H-minimal Lagrangian manifolds in Kähler manifolds. We also define the notion of Legendrian stability for these submanifolds. The Riemannian cone of a Sasaki manifold is a Kähler cone, and the converse is true. When a Sasaki manifold is regular, it is a principal S^1-bundle of a Kähelr manifold. In these situations, the H-minimality of Lagrangian submanifolds corresponds to the L-minimality of Legendrian submanifolds by taking the cone or the projection. However, there are no correspondence between the H-stability and the L-stability. In fact, we prove that any closed L-minimal Legendrian submanifolds in the odd-dimensional unit sphere is L-unstable. In contrast to this situation, we give examples of L-stable closed curves in $SL(2,\mathbb{R})$ which is the Sasakian space form with constant ϕ-sectional curvature -7.