Research plans

I will investigate the existence problem of invariant flat projective structures on homogeneous spaces. A homogeneous space G/H equipped with an invariant flat projective structure induces a Lie algebra representation of Lie(G) satisfying the same condition of infinitesimal prehomogeneous vector spaces and vice versa.

Specifically I will work on the following problems.

(1) Dimension 6 and 7

I have already classified the real Lie groups of dimension less than 6 admitting invariant flat projective and affine structures. The classification of higher dimension is itself certainly interesting, and it might give new examples of Lie algebras for the existence and non-existence of our geometric structures.

In this project we try to classify complex Lie groups of dimension 6 and 7 admitting flat complex projective or complex affine structures. The number of Lie algebras of dimension greater than 6 is very large, thus we consider the irreducible decomposition of varieties of Lie algebras. H.Takashi studied the degeneration of representations of $\mathfrak{sl}(2, \boldsymbol{C})$ to Euclidean algebra $\mathfrak{e}(2, \boldsymbol{C})$. I have almost verified that if a Lie algebra $\mathfrak{g}$ degenerates to $\mathfrak{g}'$ and $\mathfrak{g}$ admits an invariant flat projective structure, then $\mathfrak{g}$ also admits a flat projective structure again. Thus firstly I will completely prove this result.

On the other hand the irreducible components of complex Lie algebras of dimension up to 7 is determined by R.Carles and Y.Diakite. By using their list we will determine the existence and non-existence of invariant flat projective structures on a representative Lie algebra of each irreducible component. By this method I expect I can achieve the classification of dimension 6 and 7.

(2) Moduli of invariant flat projective structures

H.Tamaru, T.Hashinaga studied a moduli of left invariant metrics on solvable Lie groups and especially on dimension 3 they characterized the solsoliton metrics as submanifolds of the moduli. For my problem, the existence problem of invariant flat projective structures on homogeneous spaces, we can consider two kinds of moduli spaces. One is the moduli of invariant linear connections on homogeneous spaces. Another is the moduli of Lie algebra representations. There are so many unknown things about this problem, thus it is very challenging and interesting problem.

(3) Combinatorics of products of special linear groups

For the existence problems of invariant flat projective structures, even if we restrict our attention to only products of special linear groups, the problem is very open and difficult. For example it is shown that SL(n) admits an invariant flat projective structure by Y.Agaoka, however I have shown that $SL(2) \times SL(2)$ does not admit any invariant flat projective structure.

More generally I expect that $SL(n) \times SL(n)$ $(n > 2)$ does not admit any invariant flat projective structure. To prove this conjecture I will compute Schur functions from generally reducible representations of $SL(n) \times SL(n)$ following a method of joint work with Y. Agaoka. By using this method I already showed that $SL(3) \times SL(3)$ does not admit any invariant flat projective structure. To prove more general cases I will compute plethysm and products of Schur functions, and their decomposition by using the Littlewood-Richardson rule.