Study proposal Makiko MASE

Let F and F' be the compactifications of strange-dual pair of bimodular singularities $B = (0, f)$, $B' = (0, f')$. By a result of Ueda-M, there exist reflexive polytopes Δ, Δ' such that Δ and the polar dual Δ'^* of Δ' coincide.
Problem 1 Determine the Picard lattices $\mathrm{Pic}(\mathcal{F}_\Delta)$ and $\mathrm{Pic}(\mathcal{F}_{\Delta'})$ of the families $\mathcal{F}_\Delta$ and $\mathcal{F}_{\Delta'}$ of $K3$ surfaces associated with the polytopes Δ and Δ'.
Problem 2 Compare the Picard lattice of the family $\mathcal{F}_\Delta$ and the transcendental lattice of the family $\mathcal{F}_{\Delta'}$ in order that the families $\mathcal{F}_\Delta$ and $\mathcal{F}_{\Delta'}$ are mirror in the sense of Dolgachev.

In order to answer to Problem 1, we may need to study the method of computation of a Picard lattice of a member in a toric variety introduced by S.-M. Belcastro.

Let $\mathcal{DS}$ be the family of double sextic $K3$ surfaces, that is, $K3$ surfaces obtained by the double cover of the projective plane branched along a sextic curve.
Problem 3 Classify subfamily of $K3$ surfaces in $\mathcal{DS}$ and for each subfamily, compute the Picard lattice.

Classifying subfamily of $\mathcal{DS}$ is equivalent to classifying reflexive subpolytope of the full Newton polytope $\Delta_{(1,1,1,3;6)}$ of the weighted projective space $\mathbb{P}(1, 1, 1, 3)$ since a double sextic $K3$ surface is naturally identified with a weighted surface of degree 6 with weights $(1, 1, 1, 3)$. As to the Picard lattices, the method we used in Problem 1 can be applied to the subpolytopes that are classified.
Problem 4 For each subfamily of double sextic $K3$ surfaces in $\mathcal{DS}$, compute its mirror pair in the sense of Batyrev. Are they again subfamilies of double sextic $K3$ surfaces in $\mathcal{DS}$?
Problem 5 For general members in each subfamily of double sextic $K3$ surfaces in $\mathcal{DS}$, describe the branch locus. Which subfamilies contain general members that have structure of elliptic fibration?

Problem 4 requires a computation due to the definition of polar dual polytope. Problem 5 may interpret studies of Horikawa about the moduli space of double sextic $K3$ surfaces, and Comparin-Garbagnati about elliptic $K3$ surfaces of degree 2.
Problem 6 Let S and $\hat{S}$ be two $K3$ surfaces that are double sextic and admit elliptic fibration. Suppose S and $\hat{S}$ are mirror pair in the sense of Batyrev. Is there any relation among elliptic fibrations of S and $\hat{S}$?