Abstract of results Makiko MASE

(1) Bimodular singularities and $K3$ surfaces

Being classified by Arnold, there are singularities with two parameters. Ebeling-Ploog find a strange duality among bimodular singularities in the following sense. Let $B=(f,0)$ and $\hat{B}=(\hat{f},0)$ be two bimodular singularities, where $f=\sum_{i=1}^{3} c_i x_1^{a_{i1}} x_2^{a_{i2}} x_3^{a_{i3}}$ and $\hat{f}=\sum_{i=1}^{3} \hat{k}_i x_1^{\hat{a}_{i1}} x_2^{\hat{a}_{i2}} x_3^{\hat{a}_{i3}}$ are the defining polynomials of respective singularities. Let $\mathcal{A}=(a_{ij})$ and $\hat{\mathcal{A}}=(\hat{a}_{ij})$ be matices associated to f and $\hat{f}$. The singularities B and $\hat{B}$ are said to be strange dual if the matrices $\mathcal{A}$ and $\hat{\mathcal{A}}$ are transpose each other.

It is known that the bimodular singularities are compactifed as an anti-canonical member in the weighted projective space, and some of them are as a weighted $K3$ surface. Suppose there exists a finite group G (resp. $\hat{G}$) acting on $F=0$ (resp. $\hat{F}=0$). Denote by $\Delta_{(F,G)}$ (resp. $\Delta_{(\hat{F},\hat{G})}$) the Newton polytope of the G- (resp. $\hat{G}$-) invariants of F (resp. $\hat{F}$). We set the following question: does there exist reflexive polytopes Δ and Δ' such that Δ contains $\Delta_{(F,G)}$ and Δ' contains $\Delta_{(\hat{F},\hat{G})}$ and there exists an isometry of lattices that sends Δ to Δ'^{*}, where Δ'^{*} is the polar dual polytope of Δ'?

We get following results as an answer to this question:

Theorem 1 *Let $(F,0)$ and $(\hat{F},0)$ be compactifications of strange dual pair of bimodular singularities.*

- *(1) There exist reflexive polytopes Δ,Δ' and an isometry ϕ of lattices such that $\Delta_{(F,\{id\})}\subset\Delta$, $\Delta_{(\hat{F},\{id\})}\subset\Delta'$, and $\phi(\Delta)=\Delta'^{*}$.*
- *(2) (with K.Ueda) There exist reflexive polytopes Δ,Δ' and an isometry ϕ of lattices such that $\Delta_{(F,G)}\subset\Delta$, $\Delta_{(\hat{F},\hat{G})}\subset\Delta'$, and $\phi(\Delta)=\Delta'^{*}$, where G and $\hat{G}$ are defined by the matrices A and $\hat{A}$.*

(2) Elliptic fibrations on a singular $K3$ surface

Let X be a $K3$ surface. It is known that X admits a structure of elliptic fibration if and only if the Picard lattice $\mathrm{Pic}(X)$ of X contains the hyperbolic lattice U. It is difficult in general to determine the singular fibres of an elliptic $K3$ surface. But in case of a singular $K3$ surface, one can study elliptic $K3$ surfaces by using lattice theory. We get the following result in a study of singular $K3$ surface X with transcendental lattice $\begin{pmatrix} 2 & 0 \\ 0 & 6 \end{pmatrix}$:

Theorem 2 *There are 51 types of singular fibres of the $K3$ surface X. For some types of fibres, we get the full Mordell-Weil lattice.*