**Research Plan**

**1. Finite type invariants and 1-cocycles of virtual knots.**

In my PhD thesis, I have given a precise formulation and proof of a conjecture of M.Polyak, who predicted that a class of these invariants (called *arrow diagram formulas*) are the kernel of a linear map with values in a space of degenerate diagrams. The framework is a virtual knot theory that contains the usual knot theory in an arbitrary thickened surface.
There are two directions in which this result may reasonably be generalized.

**1.** Construct a cohomology theory for which the above map is the 0-coboundary that computes a "finite type" part of the **cohomology of virtual knot spaces**. This would be significant since very little is currently known about the topology of these spaces. I already have some serious steps achieved to understand the 1-cocycles.

Relatedly, I am currently writing programs to develop T.Fiedler's newborn theory of **quantum 1-cocycles of knots**.

**2.** Describe the whole space of **all Gauss diagram formulas** as the kernel of a map. A Gauss diagram invariant counts the subdiagrams of its variable, with *weights*. An *arrow diagram formula* is, roughly, the particular case of weights equal to the product of some parameters. One can replace that product with another elementary symmetric function: it leads to another class of finite type invariants.

Strong evidence suggest that all these classes together cover all finite type invariants. This splits the study of general Gauss diagram formulas into simple parts where the ideas from arrow diagram formulas theory may apply.

**2. Long-term projects**

Here are a few problems that I have started considering and which I would like to investigate in the future.

1. Identify the filtration of virtual finite type invariants, and a Kontsevich integral for virtual knots, inside their real versions.

2. Construct a Knot Floer Homology that categorifies T.Fiedler's 2-variable Alexander polynomial for singular links.

3. Find an algorithm to decide whether a knot in the solid torus is isotopic to a closed braid.