Plan of Research

Shin'ya Okazaki

Bridge genus and braid genus for lens space

We would like to calculate the bridge genus and the braid genus for a lens space. Let $n(p,q)$ be the number which is the minimal number of "length" of a continued fraction for $\frac{p}{q}$ with even numbers p,q. In my results, for every lens space $L(p,q)$ up to $p \leq 10$, $n(p,q)+2$ is correspond to the bridge number and the braid number of $L(p,q)$. We would like to show that the property holds for every lens space. For every orientable closed connected 3-manifold M, the following inequalities hold.

$$g_{\mathrm{H}}(M) \leq g_{\mathrm{bridge}}(M) \leq g_{\mathrm{braid}}(M).$$

Here, $g_{\mathrm{H}}(M)$ is the Heegaard genus of M. If above property holds, then we have an example of M such that $g_{\mathrm{bridge}}(M) - g_{\mathrm{H}}(M) > n$ for any n.

If a lens space $L(p,q)$ is obtained by the 0-surgery along a link L, then the component number of L is grater than or equal to 3. Thus, for the lens space obtained by the 0-surgery along L which is the closer of a pure 3-braid, the bridge number and the braid number are equal to 3. We would like to show that such lens spaces are only $L(2n,1)$.

Bridge genus and braid genus for Seifert manifold

We would like to calculate the bridge genus and the braid genus for a Seifert manifold. We can give the upper bounds for the Seifert manifolds whose base space is a 2-sphere and singular fibers are represented by an even number similarly to the calculation for a lens space. There is many Seifert manifolds represented by the braid in the table of 3-manifold by Kawauchi-Tayama-Burton. If the Seifert manifold M represented by a pure 3-braid is not obtained by the 0-surgery along any 2-component link, then the bridge genus and the braid genus of M are equal to 3. We would like to characterize the Seifert manifold M represented by the pure 3-braid.

We have already shown that $SFS[S^2 : (2,1)(2n+1,n)(4n+2,-4n-1)]$ is represented by 1^{2n}, and $SFS[S^2 : (2,1)(2n+1,n)(2n+2,-2n-1)]$ is represented by $1^{2n}, -2, 1^2, -2$. We would like to consider that the relation between the singular fibers $(2n+1,n)(4n+2,-4n-1)$ and $(2n+1,n)(2n+2,-2n-1)$ and full twists 1^{2n}.