Results of my research

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A knot is the image of an embedding of circle in the 3-sphere S^3, denoted by K. A link is the image of an embedding of circles $S^1 \cup S^1 \cup \cdots \cup S^1$ in the 3-sphere S^3. Let $L = K_1 \cup K_2 \cup \cdots \cup K_n$ be an n-component link in S^3, and $N(L)$ a tubular neighborhood of L, and $E(L)$ the exterior of L. Let $\chi(L, 0)$ be the 3-manifold obtained from $E(L)$ by attaching n solid tori $V_1, V_2, \ldots, V_n$ to $\partial E(L)$ such that the meridian of ∂V_i is mapped to the longitude of K_i $(i = 1, 2, \ldots, n)$. We call $\chi(L, 0)$ the *3-manifold obtained by the 0-surgery of* S^3 along L. It is well known that every closed connected orientable 3-manifold is obtained by the 0-surgery of S^3 along a link.

Let bridge(L) (resp. braid(L)) be the bridge index (resp. the braid index) (cf. [5]). The *bridge genus* $g_{\text{bridge}}(M)$ (resp. the *braid genus* $g_{\text{braid}}(M)$) of M is the minimal number of bridge(L) (resp. braid(L)) for any L such that M is obtained by the 0-surgery of S^3 along L. The bridge genus and the braid genus are introduced by A.Kawauchi [6].

The following is the table of the bridge genus and the braid genus of a lens space $L(p, q)$ up to $p \leq 10$.

$L(p,q)$	g_{bridge}	g_{braid}
$L(2,1)$	3	3
$L(3,1) = L(3,2)$	4	4
$L(4,1) = L(4,3)$	3	3
$L(5,1) = L(5,4)$	6	6
$L(5,2) = L(5,3)$	4	4
$L(6,1) = L(6,5)$	3	3
$L(7,1) = L(7,6)$	8	8
$L(7,2) = L(7,3) = L(7,4) = L(7,5)$	4	4
$L(8,1) = L(8,7)$	3	3
$L(8,3) = L(8,5)$	5	5
$L(9,1) = L(9,8)$	10	10
$L(9,2) = L(9,4) = L(9,5) = L(9,7)$	4	4
$L(10,1) = L(10,9)$	3	3
$L(10,3) = L(10,7)$	5	5