を満たす「捻り導分」全体の加群の関手 $\mathrm{Der}^{\sigma}(R, -)$ である．この関手は表現可能なため「捻り微分加群」が定義される．数論においては微分の代替物として $x \mapsto (x^p - x)/p$ という写像があるため，この拡張の研究には実際に価値がある ($\sigma(x) = x^p$ とおく).

2　英訳

The main goal from now is to construct a scheme theory without assuming noetherness, or a scheme theory for non-commutative rings. These subjects are in demand in the field of arithmetic geometry.

(a) **Algebraic geometry over non-commutative rings**

There are some proposals of the theory of constructing spectra of non-commutative rings; however, it is still under development at present. We know that we cannot hope for constructing spectra of non-commutative rings as a ringed space, as it may happen that the extension class may not vanish between two distinct simple modules (geometrically, this implies that there is an interaction between two distinct points).

On the other hand, for a non-commutative ring A, some may regard $\mathrm{Mod}\, A$ as a substitute of the spectrum. Indeed, when we attach with it the natural underlying functor $\mathrm{Mod}\, A \to (\mathbf{Ab})$ to the category of abelian groups, we can reconstruct A without the indefiniteness of Morita equivalence: the indefiniteness of Morita equivalence occurs when we forget this fiber functor structure. Here, we can regard a non-commutative ring as an additive category with a single object; we may hope that we can "patch" them, as in algebraic geometry, by extending this result to an appropriate class of additive categories. On the other hand, this construction is far from the construction of spectra of commutative rings; therefore we need to investigate the relation between these two constructions. There are some preceding works done by Rosenberg and others.

(b) **Extending the concept of differential modules**

Differential modules are indispensable tools to work on algebraic geometry. Differential modules can also be defined for non-commutative k-algebras, where k is a base ring, as the universal A-bimodule which satisfies the Leibniz's rule. We can also define it using the left adjoint functor of the 0-projection $(\mathbf{DGA}/k) \to (\mathbf{Alg}/k)$ from differential graded algebras to k-algebras. The relation between differential modules and formally unramifiedness naturally extends to non-commutative cases.

On the other hand, the smoothness of a k-algebra corresponds to the projectivity of modules. When we look at it carefully, smoothness is not a *local* property: some 'quasi-local' property must be assumed, such as finite-presentedness. This corresponds to the fact that we cannot determine whether an R-module M is projective by only looking at the localizations M_p, where R is a commutative ring. Therefore when we treat non-noetherian commutative rings, the criterion of smoothness corresponds to criterion of projectivity of non-finitely presented modules, which is difficult.

Also, we cannot apply the theory of differential modules directly to arithmetic geometry, as we don't have any good base ring k. As a substitute, we can consider the extended Leibniz's rule for a ring R and an endomorphism σ of multiplicative monoid on R:

$$d(ab) = \sigma(a)d(b) + d(a)b \quad (a, b \in R) \tag{1}$$

and the functor $\mathrm{Der}^{\sigma}(R, -)$ of twisted derivations d satisfying (1). This functor is representable, hence we can define a 'twisted differential module'. In arithmetics, we have a map $x \to (x^p - x)/p$ as a substitute of a differential; therefore it is worth investigating this extension. Here, we set $\sigma(x) = x^p$.