The plan of our study

This year I'm planning to study algebraic properties of quandles. The definition of a quandle is the following:

For a set X and a binaray operation $*$ on X, the pair $(X, *)$ is a quandle if the operation $*$ satisfies the following three conditions: (1) $x * x = x$ for any $x \in X$ (2) For any $x, y \in X$, there is a unique element $z \in X$ satisfying $z * y = x$. We denote $z = x \bar{*} y$. (3) For any $x, y, z \in X$, we have $(x * y) * z = (x * z) * (y * z)$. We also denote $*$ as $*^{+1}$ and $\bar{*}$ as $*^{-1}$.

Though I have read only a few papers concerning quandles, I'm impressed with it and find many questions.

Question 1. A quandle X is said to be decomposed into subquandles Y and Z, abbreviated by $X = Y \oplus Z$, if the following two conditions hold: (1) $X = Y \cup Z,\ Y \cap Z = \emptyset$, (2) $y * z = y,\ z * y = z$ $(\forall y \in Y,\ \forall z \in Z)$. Moreover, we say that a quandle X is a prime quandle, if X cannot be decomposed into two subquandles. Then we have the following question. Does any quandle have a unique decomposition into prime subquandles?

Question 2. For a quandle X, its subquandle Y is said to be normal, if we have $y * x,\ y \bar{*} x \in Y$ for any $y \in Y$ and $x \in X$.

Then, for two elements $a, b \in X$, we write $a \sim b$ if we have
$b = (\cdots((a *^{\pm} y_1) *^{\pm} y_2) *^{\pm} \cdots) *^{\pm} y_n \quad \text{for } \exists\ y_1, \exists y_2, \ldots, \exists y_n \in Y$.

We have the following questions.

(1) Is the relation $\sim$ an equivalence relation on X? If we write the quotient set as X/Y, can we introduce a quandle operation on X/Y?

(2) If X is decomposed as $X = \bigoplus_{\lambda \in \Lambda} X_\lambda$, then is the following statement right? '$X/Y = \bigoplus_{\lambda \in \Lambda} X_\lambda / (Y \cap X_\lambda)$'

Question 3. Is any quandle $(X, *)$ isomorphic to the dual quandle $(X, \bar{*})$?

I guess that these are well known results. While I continue the research on recent quandle study, I'll be able to find my own problems.