RESEARCH RESULTS

YURIKO UMEMOTO

Steinberg[1] proved that the growth series of Coxeter groups with natural generating set (Coxeter system) are rational functions. Following that, it is known the growth rates of infinite Coxeter groups, which is defined by the reciprocal of the radius of convergence of the growth series, are algebraic integers.

The group generated by the reflections with respect to bounding hyperplanes of a given Coxeter polytope in hyperbolic space is an infinite Coxeter group called a hyperbolic Coxeter group. It is already known that Salem numbers, Pisot numbers and Perron numbers appear as the growth rates of them. The following are our research results about the arithmetic nature of the growth rates.

1. Results for the growth rates of cofinite hyperbolic Coxeter groups acting on 3–dimensional hyperbolic space

In joint work with Yohei Komori at Waseda University, we proved in the papers [2, 3] that the growth rates of reflection groups with respect to Coxeter simplices or Coxeter pyramids in 3–dimensional hyperbolic space are Perron numbers, and some of them are Pisot numbers. To prove that result, we calculated the growth functions (rational functions) and derived a key property of their denominator polynomials.

2. Results for the growth rates of cocompact hyperbolic Coxeter groups acting on 4–dimensional hyperbolic space

It is known that the growth rates of cocompact hyperbolic Coxeter groups with Coxeter system acting on 2– or 3–dimensional hyperbolic space are Salem numbers or quadratic units, and that this is not true for the case 4–dimensional hyperbolic space anymore.

As a kind of generalization of a Salem number, a j–Salem number (j is a positive integer) is defined. T. Zehrt and C. Zehrt obtained infinite families of cocompact hyperbolic Coxeter groups whose growth functions have denominator polynomials with a distribution of roots similar to the one of the minimal polynomial of a 2–Salem number, by constructing the Coxeter garlands in 4–dimensional hyperbolic space. Inspired by their work, we construct in [4] infinite families of cocompact hyperbolic Coxeter groups by building Coxeter dominoes in 4–dimensional hyperbolic space. Coxeter dominoes are Coxeter polytopes obtained by gluing together finitely many pieces of a compact totally truncated Coxeter simplex, given by Schlettwein, along the isomorphic orthogonal facets. We also prove that the growth rates of a certain infinite subsequence of Zehrt's Coxeter garlands are 2–Salem numbers.

References

[1] R. Steinberg, Endomorphisms of linear algebraic groups, Memoirs of the American Mathematical Society **80**, American Mathematical Society, 1968.

[2] Y. Komori, Y. Umemoto, On the growth of hyperbolic 3-dimensional generalized simplex reflection groups, Proc. Japan Acad., 88, Ser. A (2012), pp. 62–65.

[3] Y. Umemoto, The growth rates of non-cocompact 3-dimensional hyperbolic Coxeter tetrahedra and pyramids, proceedings of the 19th ICFIDCAA Hiroshima 2011, Tohoku University Press (2012), pp. 261–268.

[4] Y. Umemoto, The growth function of Coxeter dominoes and 2–Salem numbers, to appear in Algebraic and Geometric Topology.