

これまでの研究成果のまとめ (英訳)

My past work was devoted to investigate the arithmetic geometry concerning hyperbolic (algebraic) curves. I shall explain concretely the results in my past works as follows.

① Cuspidalization problem for hyperbolic curves over finite fields

In the paper [W1], I study some group-theoretic constructions, so called “cuspidalization problem”, associated to arithmetic fundamental groups of hyperbolic curves over finite fields. As one of the main results of the paper, I obtained a certain group-theoretic construction of geometrically pro- l fundamental groups of configuration spaces of hyperbolic curves over finite fields. It contains a perfect generalization of the work of S. Mochizuki and Y. Hoshi.

② Joshi’s conjecture: an explicit formula for the number of dormant indigenous bundles

A dormant indigenous bundle is an integrable projective line bundle on a proper hyperbolic curve of positive characteristic satisfying certain conditions. Dormant indigenous bundles were introduced and studied in the p -adic Teichmüller theory developed by S. Mochizuki. K. Joshi proposed a conjecture concerning an explicit formula for the number of dormant indigenous bundles on a general curve. In the paper [W2], I gave a proof of this conjecture of Joshi.

③ Combinatorics of dormant indigenous bundles (Liu-Osserman conjecture)

It follows from work of S. Mochizuki, F. Liu, and B. Osserman that there is a relationship between Ehrhart’s theory concerning rational polytopes and the geometry of the moduli stack classifying dormant indigenous bundles on a proper hyperbolic curve in positive characteristic. In the paper [W3], by applying the result of ②, I explored further this nontrivial interaction, and presented some explicit computations of invariants associated with the objects involved. Moreover, I gave a proof of a conjecture proposed by F. Liu and B. Osserman.

④ The symplectic nature of dormant indigenous bundles and their moduli stack

In the paper [W4], I obtained a certain compatibility of the canonical symplectic structures, defined on moduli stacks concerning dormant indigenous bundles. Also, in the paper [W5], I succeeded to construct a canonical deformation quantization satisfying a certain nice condition by means of a dormant indigenous bundle. These results may be thought of as analogues in positive characteristic of the past results in the theory of compact hyperbolic Riemann surfaces.