「今後の研究計画」の英訳

I shall state my future plan for the current research. For the study of asymptotic behavior of solutions to the scalar conservation law with the linear viscosity, Matsumura-Yoshida(2012) investigated the case where the flux function is convex but linearly degenerate on some interval, and showed that the solution tends toward the multiwave pattern consists of the viscous contact wave and the rarefaction waves, not only for the Cauchy problem but also for the initial-boundary value problem. However, except for that case, it remains that the case where the multiwave patterns consist the stationary solution, so-called the boundary layer solution which appears by the boundary condition if we consider the initial-boundary value problem. In particular, it is technically difficult to prove that asymptotic stability for the case where the multiwave pattern consists the boundary layer solution and the viscous contact wave. The main reason for the difficulty is that it is clear that the interaction itself contributes to the solvability of the equation more slightly for the boundary layer solution and the rarefaction wave than for the rarefaction wave and the viscous contact wave. However we recently succeeded in illuminating the nonlinear interaction of these waves, and establishing new method for mixing it with the energy norm of the solution. I am now trying to solve the problem by employing the method. Furthermore, on the initial-boundary value problem, it must be the most difficult problem that the case where the multiwave pattern consists three nonlinear waves, that is, the boundary layer solution, the rarefaction wave and the rarefaction wave. I will try to solve the problem by investigating the interaction of the three waves. After that, we will obtain its precise time-decay estimates.

We state the study for the asymptotic behavior of solutions to the scalar conservation law with the nonlinear viscosity. This case for the viscosity is much more complex and difficult to treat the equation than the linear case. For the initial-boundary value problem, although the flux function is uniformly convex, there has been no results even for the asymptotic stability. For that case, we can expect the asymptotic stability of the stationary solution and the single rarefaction wave and I will obtain them. We have already gotten the energy estimate for the lower order derivative, to be necessary for solving thoese problems. However, what is more important to solve them is the energy estimate for the higher order one. We also note that the previous arguments by Matsumura-Nishihara(1994, Nonlinear Anal. TMA) and Yoshida(submitted) are not applicable to get the higher order one. To do that, we expect we can obtain them by making use of the shape of the equation, and constructing the weighted energy estimate depending on the waves as the asymptotic state. Further, similar to the linear viscosity case, we can expect the asymptotic stability of the boundary layer solution which appears by the boundary condition, and I hope to obtain them. In addition, we also hope to investigate the stability of the superpositions of the rarefaction wave, the boundary layer solution or the viscous contact wave (constructed by the Barenblatt solution to the porous medium equation). Also for the Cauchy problem, there exists the case where the asymptotic state can be expected the single shock wave under the far field states and the flux function which are suitably chosen. I hope to investigate it.

We finally state the study for the asymptotic behavior of solutions to the Korteweg-de Vries-Burgers equation. Under considering asymptotic behavior of solutions, the most difficulty in treating this equation might be how to estimate the dissipation term by making use of the properties for the nonlinear waves. For the Cauchy problem or the initial-boundary value problem, there has been no results for the asymptotic stability, in particular the multiwave pattern, except those of the single rarefaction wave and the single shock wave. For the Cauchy value problem, it must be an important problem that the asymptotic stability of the multiwave pattern of the rarefaction waves and a similarity solution (viscous contact wave in a wide sense) corresponding the viscous contact wave. Furthermore, we can expect that the more variational and complex multiwave patterns will appear, such as the superpositions constructed by the rarefaction wave, the viscous contact wave, the boundary layer solution and so on. Therefore, I hope to solve all of them.