「これまでの研究成果のまとめ」の英訳

I have been investigating the asymptotic behavior of solutions to the Cauchy problems for the scalar viscous conservation law. In particular, I study the case where the flux function is convex but linearly degenerate on some interval.

For the viscous conservation law with the linear viscosity term, it has been known that the asymptotic behavior of the solution can be characterized by the global weak solution (``Riemann solution'') of the corresponding Riemann problem for the non-viscous conservation law. In fact, when the flux function is convex (``genuinely nonlinear'') on the whole space, Il'in-Oleĭnik(1960) showed that if the Riemann solution consists of a single rarefaction wave solution or a single shock wave (``Lax shock wave'') solution, then the global solution in time of the Cauchy problem tends toward the rarefaction wave solution of the hyperbolic part or the corresponding smooth traveling wave solution (``viscous Lax shock wave'') respectively. For general flux functions which are not uniformly genuinely nonlinear, there had been no results until Matsumura-Nishihara(1994, Comm. Math. Phys.) studied the case where the Riemann solution consists of a single shock wave (``Oleĭnik shock wave'') solution and showed the asymptotic stability of the corresponding ``viscous Oleĭnik shock wave'' solution. Matsumura-Yoshida(2012) investigated the case where the flux function is convex but linearly degenerate on some interval, and succeeded in showing that if the Riemann solution consists of rarefaction waves and contact discontinuity then the solution of the Cauchy problem tends toward the linear combination of the rarefaction waves and the corresponding viscous contact wave of the Cauchy problem as the time goes to infinity. It is the first result concerning with the asymptotics toward a multiwave pattern of the Cauchy problem for the scalar viscous conservation law. Furthermore, Yoshida(2014) obtained the decay rate estimates of the asymptotics toward the multiwave pattern.

Now let us turn to the Cauchy problem for more subtle case of the viscous conservation law with the p-Laplacian type viscosity. On the asymptotic behavior in time of the solution to the Cauchy problem, there has been no results except one by Matsumura-Nishihara(1994, Nonlinear Anal. TMA) where they studied the case where the flux function is genuinely nonlinear and showed the asymptotic stabilities of the far field constant state (trivial solution) and the single rarefaction wave. Yoshida(submitted) succeeded in extending the results for the linear viscosity case even to the case of the p-Laplacian type viscosity. Namely, when the flux function is convex but linearly degenerate on some interval and the Riemann solution consists of rarefaction waves and contact discontinuity, then the solution of the Cauchy problem tends toward the linear combination of the rarefaction waves and the corresponding viscous contact wave (constructed by the Barenblatt solution of the porous medium equation) as the time goes to infinity. In this case, it should be emphasized that since both the rarefaction wave and the viscous contact wave are nonlinear ones, more subtle estimates for the nonlinear interaction of waves are needed than the case of linear viscosity.

On the decay rate in time of the solution to the Cauchy problem, there has been no results for the conservation law with the p-Laplacian type viscosity. Under the case where the flux function is genuinely nonlinear, Yoshida(submitted) obtained the almost optimal time-decay estimates for the stability of the far field constant state and the single rarefaction wave by Matsumura-Nishihara. Furthermore, Yoshida(submitted) also obtained the almost optimal time-decay estimates for the one of the multiwave pattern of the rarefaction waves and the viscous contact wave.

Finally, we state the study for the dissipative wave equation. On the asymptotic behavior in time of the solution, there has been no results for the Cauchy problem except few results of the single shock wave or the superposition of the rarefaction wave and the stationary solution for the initial-boundary value problem. Yoshida(in preparation) investigated the case where the flux function is convex but linearly degenerate on some interval, and succeeded in showing that the solution tends toward the similar multiwave pattern pattern of the Cauchy problem for the scalar conservation law with the linear viscosity, that is, superposition of the rarefaction waves and the viscous contact wave. We also obtained the similar asymptotic stability for the initial-boundary value problem.