Research planning

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1. Jacobi's inversion problem for telescopic curves

Let X be an algebraic curve of genus g and $du = {}^t(du_1, \ldots, du_g)$ a basis of the vector space consisting of holomorphic one forms on X. We take a point P_0 on X. For $\xi = {}^t(\xi_1, \xi_3, \ldots, \xi_{2g-1}) \in \mathbb{C}^g$, we want to express $(P_1, \ldots, P_g)$ such that

$$\sum_{i=1}^{g} \int_{P_0}^{P_i} du = \xi$$

in terms of ξ. This problem is called Jacobi's inversion problem. For (n,s) curves, Matsutani and Previato [2] solved the problem by using the sigma functions. The applicant will consider the problem for telescopic curves, which contain (n,s) curves.

2. Inversion of algebraic integrals

Let X be a hyperelliptic curve defined by $y^2 = 4x^{2g+1} + \lambda_{2g}x^{2g} + \cdots + \lambda_0$ and $du = {}^t(du_1, \ldots, du_g)$ a basis of the vector space consisting of holomorphic one forms on X. We take a point P_0 on X. For $\xi = {}^t(\xi_1, \xi_3, \ldots, \xi_{2g-1}) \in \mathbb{C}^g$, we want to express $P = (x, y)$ such that

$$\int_{\infty}^{P} du = \xi$$

in terms of ξ. For $g = 2$, it is known that $x = -\frac{\sigma_3(\xi)}{\sigma_1(\xi)}$. For the sigma function $\sigma(u)$ and $u = {}^t(u_1, u_3, \ldots, u_{2g-1})$, we set $\sigma_{i_1,\ldots,i_k}(u) = \frac{\partial^k}{\partial u_{i_1} \cdots \partial u_{i_k}} \sigma(u)$. For general cases, it is conjectured that $x = -\frac{\sigma_{2g-1,1^M}(\xi)}{\sigma_{2g-3,1^M}(\xi)}$ with $M = \frac{1}{2}(g-2)(g-3)$ [1]. The applicant tries to solve the conjecture and consider the inversion of algebraic integrals for more general curves.

3. Dimension of moduli space of telescopic curves

Telescopic curves are algebraic curves determined by a sequence of positive integers $(a_1, \ldots, a_m)$ which satisfies a condition. For example, if $m = 2, a_1 = n, a_2 = s$, then telescopic curves are defined by $y^n = x^s + \sum_{in+js<ns} \lambda_{ij} x^i y^j$ $(\lambda_{ij} \in \mathbb{C})$, which is called (n,s) curves. The applicant will consider the dimension of moduli space of telescopic curves.

4. Condition for the fact that the maximum of the gap value is $2g-1$

The numbers that are not contained in a monoid generated by relatively prime positive integers $(a_1, \ldots, a_m)$ are called gap values. If $(a_1, \ldots, a_m)$ is telescopic, then the maximum of the gap values is $2g-1$ (g is the number of gap values). Conversely, the applicant will consider whether $(a_1, \ldots, a_m)$ is telescopic if the maximum of the gap values is $2g-1$. Miura curves are algebraic curves determined by relatively prime positive integers $(a_1, \ldots, a_m)$. Telescopic curves are Miura curves determined by $(a_1, \ldots, a_m)$ satisfying the telescopic condition. The fact that the maximum of the gap values is $2g-1$ is essential to prove the algebraic nature and the addition formulae of the sigma functions. Therefore, it is important for extending the sigma functions to the case of Miura curves which are more general than telescopic curves to consider the condition for the fact that the maximum of the gap values is $2g-1$.

References

[1] V. Enolski et al., J. Math. Phys. 53, 012504 (2012).

[2] S. Matsutani and E. Previato, J. Math. Soc. Japan, 60 (2008) 1009-1044.