

Research Results

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The holonomy groups of simply-connected Riemannian manifolds were classified into seven kinds of groups by Berger. There are $SU(n)$, $Sp(n)$, $Sp(n)Sp(1)$, G_2 and $Spin(7)$ as a special holonomy group. In particular, a Calabi-Yau manifold is defined as a Kähler manifold satisfying that the holonomy group is the subgroup of a special unitary group $SU(n)$. Using a calibration, Harvey and Lawson many example of constructed homologically volume minimizing submanifolds in the Riemannian manifold which have special holonomy groups. In the case of complex n dimensional Calabi-Yau manifold, the real part of complex volume form is calibration and a calibrated submanifold is called a special Lagrangian submanifold. Strominger, Yau and Zaslow suggested that, from view point of geometry, the mirror symmetry between Calabi-Yau 3-folds should be explained in terms of dual fibrations by special Lagrangian 3-tori. Today many mathematicians pay attention to special Lagrangian submanifold.

Stenzel constructed Ricci-flat Kähler metrics on the cotangent bundle of compact rank one symmetric spaces with cohomogeneity one action. Ionel and Min-Oo studied special Lagrangian submanifolds in the cotangent bundle of 3-sphere invariant under the 2-torus. Anciaux studied special Lagrangian submanifolds in the cotangent bundle of n -sphere invariant under $SO(n)$. As a generalization of the above results, we classify cohomogeneity one special Lagrangian submanifolds in the cotangent bundle of the sphere S^n invariant under $SO(p) \times SO(q)$ ($p+q = n+1$) ([1]). First we construct Lagrangian submanifolds by the moment map technique. Since these Lagrangian submanifolds are of cohomogeneity one, the condition to be special Lagrangian is reduced to certain ordinary differential equations. We analyze the solutions of the ODE, and investigate the asymptotic behavior and singularities of the corresponding special Lagrangian submanifolds.

It is known that every homogeneous hypersurfaces in sphere is an orbit of s -representation of Riemannian symmetric spaces of rank two by Hsiang and Lawson, etc. In several cases we construct cohomogeneity one special Lagrangian submanifolds in the cotangent bundle of the sphere in my thesis.