# Summary of research results

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I have studied internal spaces preserving supersymmetries in string compactification. Especially, I have invetigated supersymmetric solutions in supergravity theories in the case of 6-, 7-, 8-dimensional internal spaces, where an $SU(3)$-structure on 6-dimensional manifolds, a $G_2$-structure on 7-dimensional manifolds, and an $Spin(7)$-structure on 8-dimensional manifolds play an important role.

In 7 dimensions, we introduced a class of $G_2$-structure associated with the Abelian heterotic supergravity theory. The class has a closed 3-form torsion $T_7$ and an exact Lee form $\Theta_7$. Hence we identified the flux $H_7$ with $T_7$ and the dilaton $\varphi_7$ with $\Theta_7$. The class was determined by the choice of $(g_7, \Omega, F_7)$, where $g_7$ was the metric associated with the fundamental 3-form $\Omega$ and $F_7$ was the field strength of $U(1)$ gauge field. In this theory, the flux $H_7$ and the field strength $F_7$ should satisfy the Bianchi identity, $dH_7 = F_7 \wedge F_7$. In addition, $F_7$ should satisfy the generalized self-dual equation, $*(\Omega \wedge F_7) = F_7$, which arises from $G_2$ irreducible representation. The defining equations of a class on the cohomogeneity one manifold of the form $\boldsymbol{R}_+ \times S^3 \times S^3$ became first order ordinary differential equations. We obtained the formulae which give the $S^3$-bolt solutions from regular solutions of the Ricci-flat $G_2$ equations, and $T^{1,1}$-bolt solutions were also obtained numerically. In 8 dimensions, we introduced a class of $Spin(7)$-structure associated with the Abelian heterotic supergravity theory. The class was determined by the choice of $(g_8, \Psi, F_8)$, where $g_8$ is a metric associated with the fundamental 4-form $\Psi$ and $F_8$ is the field strength of $U(1)$ gauge field. We assumed the manifold $\boldsymbol{R}_+ \times M_{3-Sasaki}$, where $M_{3-Sasaki}$ denotes a manifold with a 3-Sasakian structure, and then defining equations reduced to first order ordinary differential equations. The regular solutions were obtained from the Ricci-flat $Spin(7)$ metrics.

In dimension 6, we constructed an intersecting metric $g_6$ by superposing two Gibbons–Hawking metrics with the conformal factors, i.e., HKT metrics. The metric $g_6$, the fundamental 2-form $\kappa$, and the complex $(3,0)$-form $\Upsilon$ satisfy the defining equations of the $SU(3)$-structure associated with the NS sector in $E_8 \times E_8$heterotic supergravity, where the Lee form $\Theta_6$ is an exact 1-form and the Bismut torsion $T_6$ is closed 3-form. We identified the flux $H_6$ with the torsion $T_6$, the dilaton $\varphi_6$ with $\Theta_6$, and the field strength $F_6$ with Hull curvature $R^-$. The solution has 2-dimensional harmonic functions $\phi$, $\tilde{\phi}$, $\Phi$, and $\tilde{\Phi}$; however, the condition $\phi = \tilde{\phi} = \Phi = \tilde{\Phi}$ is required such that the metric is non-negative and the dilaton takes real value. The manifold $(M_6, g_6, \kappa, \Upsilon)$ is a Calabi–Yau with torsion manifold, and thus, $(g_6, H_6, \varphi_6)$ are supersymmetric solutions in the theory. Thus, the solution is characterized by a pair of harmonic functions $(\phi, \psi)$, which are related by Cauchy–Riemann equations. To construct the solutions which break $E_8$ to $SO(10)$, one need to obatin the solution which has $SO(6)$ Hull holonomy. However, the holonomy group of the obtained manifold is an $SO(4)$. To recover the $SO(6)$ holonomy, we obtained the supersymmetric solution $(\tilde{g}_6, \tilde{\varphi}, \tilde{H})$ with the $SO(6)$ holonomy of the Hull connection by T-dualizing along $\partial_3$ and then $\partial_5$. To obtain the compact 6-dimensional space, we chose $\phi + \sqrt{-1}\psi$ as the Weierstrass's $\wp$ function. Then, the solution had codimension-1 singularity hypersurfaces.