

## Plan of study

1. Classification of homogeneous Lagrangian submanifolds in Kähler manifolds and the H-stability.

It is an important problem in symplectic geometry and differential geometry to classify homogeneous Lagrangian submanifolds in Kähler manifolds. For instance, any compact homogeneous Lagrangian submanifold is H-minimal, and the Hamiltonian stability is investigated by the harmonic analysis. Recently, Bedulli-Gori classified all compact homogeneous Lagrangian submanifolds in  $\mathbb{C}P^n$ , and Ma-Ohnita done in  $Q_n(\mathbb{C})$  by using the moment maps. Moreover, Ma-Ohnita decided the H-stability of these submanifolds in  $Q_n(\mathbb{C})$ . When the case of  $\mathbb{C}P^n$ , Oh and Ohnita posed the problem: "Is any embedded compact minimal Lagrangian submanifold in  $\mathbb{C}P^n$  H-stable?" To consider this problem, we want to investigate the H-stability of the homogeneous Lagrangian submanifolds in  $\mathbb{C}P^n$ . The calculation of Maslov number of these submanifolds is also a basic problem in symplectic geometry.

Y. -G. Oh conjectured that "any Einstein real form in a Kähler-Einstein manifold is Hamiltonian volume minimizing". To consider this conjecture, we investigate Lagrangian submanifolds in the homogeneous Kähler-Einstein manifolds, i.e., the complex flag manifolds. First, we assume the Kähler-Einstein structure on complex flags compatible with the standard complex structure, and investigate geometric properties of Lagrangian submanifolds.

2. Construction of H-minimal Lagrangian submanifolds in the complex hyperquadric.

F. Hélein and P. Romon showed that the non-linear partial equation of Hamiltonian minimal Lagrangian surfaces in a Hermitian symmetric space of complex dimension 2 is an integrable system, and proved that the DPW method can be applied for these surfaces. In fact, Hélein-Romon and H. Ma give a Weierstrass formula for H-minimal Lagrangian surfaces in  $\mathbb{C}^2$  and  $\mathbb{C}P^2$ . Moreover, they also give a classification of H-minimal Lagrangian tori. In this study, we construct an H-minimal Lagrangian surface in the complex hyperquadric by using their method.