

The conformal classes of a 1-form α and a 2-form ω on a $(2n + 1)$ -manifold M^{2n+1} determine an almost contact structure if $\alpha \wedge \omega^n$ is a volume form. If α is a contact form, we may put $\omega = d\alpha$. On the other hand, if α defines a codimension one foliation, the restriction $\omega|_{\ker \alpha}$ defines a corank one almost Poisson structure. If moreover $\alpha \wedge d\omega = 0$, it is a Poisson structure. The notion of confoliation due to Eliashberg and Thurston is improved in [11] to an intermediary almost contact structure between these extreme cases. The new version of [11] contains a further reformed notion based on Weinstein's formulation of nearly Poisson structure found in physics. Mitsumatsu achieved an example of corank one Poisson structure on S^5 . The standard contact structure ξ_0 on S^5 can be deformed into Mitsumatsu's Poisson structure via almost contact confoliations ([11]). Recently an infinite family of such deformations of ξ_0 into distinct Poisson structures, indicated by triples of positive integers, were constructed in [13]. Also a corank one Poisson structures on $S^4 \times S^1$ was constructed in [12].

Bennequin showed the relative Thurston inequality for any surface with contact-type boundary in the standard S^3 . Eliashberg proved the same inequality on any non-overtwisted contact 3-manifold, and recently completed his h-principle saying that any almost contact structure in any (odd) dimension deforms into a unique contact structure with a certain "disk" along which the structure is overtwisted. Giroux proved any surface in a contact 3-manifold is approximated by a "convex" one. In [10], a hypersurface with contact-type boundary in the standard S^{2n+1} ($n > 1$) was constructed so that it violates the above inequality and is far from convex. It was conjectured that the inequality holds for any convex hypersurface in S^{2n+1} . A modification of contact structure was introduced in [9] which is in a sense a generalization of Lutz twist (called Lutz-Mori twist in some literatures). In $S^5(\subset \mathbb{C}^3)$, one can perform it along a link of certain complex surface singularities to produce a convex hypersurface violating the inequality. Niederkrüger et al. proved that this modification spoils the symplectic null-cobordancy of S^5 .

In [4], a "spinning" immersion of a given contact 3-manifold into the standard S^5 was constructed. Martínez Torres generalized it to a spinning immersion of contact $(2n + 1)$ -manifold into the standard S^{4n+1} . In [8], a deformation of the standard $S^3 \subset S^5$ is constructed so that the limit is the union of Legendrian submanifolds of S^5 which is the leaves of the Reeb foliation of S^3 . The toric method used there is applied by Naohiko Kasuya in his study on cusp singularities.

An early result in [3] on deformation of 3-dimensional contact structure into foliation implies that many foliations with Reeb components satisfy the inequality in contrast to the confoliation theory. [7], [6] and [5] contains relevant results concerning homological overtwistedness, Dehn fillings and Bennequin's lemma.