# Research project

An interesting subject that we can find in random dynamical systems would be the stabilization of some dynamical property under small perturbations in a family of dynamics with parameters exhibiting different properties, called *noise-induced phenomena*. Although most of what is known about noise-induced phenomena come from numerical experiments and have not been mathematically proven, rigorous result exist, e.g., the stabilization of the chaotic property of unimodal mappings under small noise, which has been started by Katok and Kifer in 1986. The past research [1] also was started with interest in this context. I am generally interested in these noise-induced phenomena, and specifically has the following plans for future research.

## Partially expanding maps

Two problems related with the study in [1] are considered as follows. Firstly, an extension of the genericity result in [1] mentioned above to nonlinear settings is considered. Recently some result is obtained in a collaborative work with Masato Tsujii and Jens Wittsten, and its manuscript is in preparation. This paper is also written with the intention of simplifying the complicated proof in Tsujii's work in 2008. Secondly, by preparative works mentioned above and in [1], I believe that even if the unperturbed map does *not* satisfy the transversality condition, one can prove that perturbed systems admit a unique aceip whose correlation functions decay exponentially fast if the perturbation scheme is "absolutely continuous and strongly mixing". Since partially expanding maps which do not satisfy the transversality condition are not even weakly mixing, this can be interpreted as a new noise-induced phenomenon.

## Mañé's dichotomy/Singer's theorem

In 1985 Mañé showed the dichotomy that "any one-dimensional dynamical system on an invariant set which does not include any critical point, sink, or neutral periodic point is either hyperbolic or conjugate to irrational rotation." When I naively inferred an extension of this result to random one-dimensional dynamical systems, then it was (only) conjectured that the noise parameter space is (up to zero measure set) the union of a parameter set in which the dynamics is hyperbolic and a parameter set in which the dynamics is conjugate to irrational rotation. However, it was soon realised that due to the randomness of our dynamics we possibly obtain a stronger result "each of these parameter sets is either measure 0 or 1", or conversely, a possibility that "the third phenomena" is observed still remains. As a first step to solving this problem, I have been studying an extension/counterexample of Singer's theorem ("the number of non-expanding periodic points of a one-dimensional dynamical system with negative Schwarzian derivative is bounded by the number of critical points") to random one-dimensional dynamics. Initial attempts indicate that the following version of Singer's theorem seems true: the basin of the (semi-)attractor of each "periodic" graph with a non-positive Lyapunov exponent and each connected component of the critical set always have an intersection with positive measure; on the other hand, one can construct an example of a dynamics with infinitely many attractors in spite of that the connected components of the critical set are finitely many.