Past research

In dynamical systems theory, two questions have been studied for a long time; one is how the typical orbits asymptotically behave, and the other is how stable the dynamical behaviour is under perturbations of the systems. I have obtained some results on the stability of statistical behaviour of two broad classes of dynamical systems — *expanding maps* and *partially expanding maps* — under small noise perturbations, which is called *stochastic stability*.

Stochastic stability of partially expanding maps

Stochastic stability was introduced as the stability of statistical properties — in our setting, the existence of an *absolutely continuous ergodic invariant probability measure* (abbreviated *aceip*) and *exponential decay of correlation functions* — under small perturbations by Kolmogorov and Sinai in 1972. In the context of random perturbations of hyperbolic maps (i.e., dynamics with exponential expanding/contracting directions) by Markov chains, the theory was basically completed by Kifer in the 80s.

In contrast to the hyperbolic case, only a few stochastic stability results are known for dynamical systems with nonhyperbolic directions (such as hyperbolic fows or partially hyperbolic maps). The difficulty of nonhyperbolicity had already appeared in the study of unperturbed systems; for instance, the existence of aceip of suspension semifows of linear expanding maps whose correlation functions decay exponentially fast was first proven in 2008 by Tsujii. He introduced a generic condition (called the *transversality condition*) and showed the exponential decay of correlation functions by carrying out functional analysis under this condition. By developing Tsujii's argument further, I and Jens Wittsten showed in [1] that if a U(1)-extension of a linear expanding map on the circle satisfies *partial captivity*, then the dynamics admits a *unique* aceip (the existence of aceip's had been proven by Faure in 2012) whose correlation functions decay exponentially fast, and that partial captivity is stronger than transversality, but still generic as Faure conjectured. Furthermore, in the paper it was shown that under small noise perturbations the generic dynamics preserves the unique existence of aceip whose (quenched) correlation functions decay exponentially fast, and that the aceip of randomly perturbed dynamics converges to the aceip of the unperturbed dynamics when the noise level goes to zero. To the best of our knowledge, this is the first result for stochastic stability of partially expanding maps.

Stochastic stability under non-invertible perturbations

Stochastic stability is a rather vague notion depending on the nature of the dynamical systems under consideration and subjects of interest. A standard formulation is through introducing skew-product mappings with base transformations preserving the measure on noise space (whose fibre component is called *random dynamical systems*, abbreviated *RDSs*). A significant restriction on known results with perturbations induced by skew-product mappings (including the result [1] mentioned above) is that the base transformations are required to be invertible. I demonstrated in [2] stochastic stability for expanding maps under perturbations induced by skew-product mappings whose base dynamics are not invertible necessarily.