Research Plan

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My graduate research gave me basic knowledge on the existence, regularity and asymptotic behavior as well as the uniqueness of the Navier-Stokes equation. Based on this research experience, I am planning the further study of the Navier-Stokes equation, especially the stationary equations in 2D exterior domains. Here I briefly describe two research projects. In addition to the following projects, the existence of a stable solution whose first derivative decays like $|x|^{-2}$ at infinity is also an interesting problem. The analysis of the stationary problems in other 2D and 3D unbounded domains is also my research subject. Furthermore, I would like to challenge the time-periodic problem for the nonstationary Navier-Stokes equation by making use of the knowledge obtained in the joint works with Y. Taniuchi and R. Farwig,.

1. Ill-posedness of the 2D exterior problem.

Owing to its peculiar difficulty such as the Stokes paradox, the stationary Navier-Stokes equations in 2D exterior domains are known as a challenging problem. Since the linear approximation is not useful in general, the nonlinearity is expected to contribute to the good property of solutions at infinity. However, many mathematicians believe that the contribution of the nonlinearity would not guarantee the decay of solutions and that a solution decaying at infinity exists only if special compatibility condition is satisfied. I predict that we cannot expect to obtain a solution without assuming special conditions such as symmetry. The goal of this project is to give a proof of this conjecture. As a first step in this project, I plan to show that if a solution with finite Dirichlet integral belongs to some L^p space, then it must satisfy the special compatibility condition. I have already observed that this problem has a positive answer if $p \leq 4$ and, with the detailed analysis of properties of the solution, I would be devoted to the extension to the case $p > 4$.

2. Asymptotic property of stationary solutions.

The difficulty of the 2D exterior problem for the stationary Navier-Stokes equation lies in the behavior of solutions at infinity and we know very little about the asymptotic behavior of solutions with finite Dirichlet integral. Gilbarg-Weinberger showed that a solution u with finite Dirichlet integral converges to some constant vector u_c at infinity, however, it is still an open problem whether u_c coincides with the prescribed vector u_∞. Furthermore, their method is available only when there is no external force. Indeed, one of the important steps in their proof is to apply the maximum principle to the linear elliptic equation of second order, which the quantity $\Phi := \frac{1}{2}|u|^2 + p$ consisting of the velocity vector u and the pressure p satisfies. However, if the external force is not zero, the elliptic equation contains a term relevant to the external force and it prevents us from applying the maximum principle. The aim of this project is to extend the result of Gilbarg-Weinberger, the convergence of a solution at infinity, to the case where nonzero external force exists. My approach to this problem is based on the detailed analysis of the existence and asymptotic behavior of the second order elliptic equation, containing a term of first order whose coefficient is the function u, mentioned above. This approach would help us overcome the difficulty stated above, and I would also consider challenging the problem on the coincidence of the vectors u_c and u_∞.