## Results of my research

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**Definition of handlebody-knot and earlier study**

A handlebody-knot is a handlebody embedded in the 3-sphere $S^3$, denoted by $H$. A fundamental problem in handlebody-knot theory is the classification problem of handlebody-knots. In the paper [Ishii– Kishimoto–Moriuchi–Suzuki], we have the table of genus two handlebody-knots up to six crossings. In this paper, the table obtained by using a computer, lists the number of conjugacy classes of representations from some subgroup of $SL(n, \mathbb{Z})$ to $G(H)$, and the quandle coloring invariant. Here, $G(H)$ is the fundamental group of the exterior of $H$.

**Results of my research**

We obtain a new invariant of handlebody-knots comes from the Alexander invariant. The Alexander polynomial is an invariant for a pair of a handlebody-knot and its meridian system. However, the new invariant depend on only the handlebody-knot type.

For convenience, we suppose that the genus of a handlebody-knot is two. We choice a meridian system $M$ for given handlebody-knot $H$. In general, the second multi-variable Alexander polynomial for the pair $(H, M)$ is non-trivial, denoted by $\Delta_{(H,M)}(s,t) = \Sigma c s^x t^y$. We have the graph from $\Delta_{(H,M)}(s,t)$ as follows.

For each term $T_i = c_i s^x t^y$ of $\Delta_{(H,M)}(s,t)$, we take the black vertex $b_i$ of the graph labeled by $c_i$. For any three terms $T_i = c_i s^{x_i} t^{y_i}$, $T_j = c_j s^{x_j} t^{y_j}$ and $T_k = c_k s^{x_k} t^{y_k}$, we take three points $(x_i, y_i)$, $(x_j, y_j)$ and $(x_k, y_k)$ in $\mathbb{R}^2$, respectively. We take the white vertex of the graph labeled by $x_i y_j + x_j y_k + x_k y_i - x_i y_k - x_j y_i - x_k y_j$ which is connected to three black vertex $b_i$, $b_j$ and $b_k$. Then, we have the graph, denoted by $G_H$.

Theorem

The graph $G_H$ is an invariant of $H$.

# References

[1] A. Ishii, K. Kishimoto, H. Moriuchi and M. Suzuki, The table of genus two handlebody-knots up to six crossings, *J. Knot Theory Ramifications.*, Vol. 21, No. 4 (2012), 1250035, 9 pp.