Brief summary on the study

The author have studied the elliptic Ding-Iohara-Miki algebra and related materials. First we explain how the elliptic Ding-Iohara-Miki algebra arises. In 1997, Ding and Iohara studied generalizations of the quantum affine algebra $U_q(\widehat{sl_2})$ called the Ding-Iohara algebras. In 2007, Miki introduced a q-analog of $W_{1+\infty}$ algebra which has the structure of the Ding-Iohara algebra. In the following, we call the Miki's quantum algebra the Ding-Iohara-Miki algebra. In 2009, Feigin-Hashizume-Hoshino-Shiraishi-Yanagida have indicated that the Ding-Iohara-Miki algebra arose from the free field realization of the Macdonald operator. They have also constructed two commuting families of q-difference operators containing the Macdonald operators by the free field realization and the trigonometric Feigin-Odesskii algebra. Here the trigonometric Feigin-Odesskii algebra is a trigonometric degeneration of the elliptic Feigin-Odesskii algebra which was originally introduced by Feigin and Odesskii. In addition, there exists an elliptic analog of the Macdonald operator called the elliptic Ruijsenaars operator. Feigin-Hashizume-Hoshino-Shiraishi-Yanagida have made an attempt to construct two sets of q-difference operators containing the elliptic Ruijsenaars operator by using the elliptic Feigin-Odesskii algebra. However some problems on the free field realization of the elliptic Ruijsenaars operator arose, thus the program was not complete.

To solve the above problem, the author paid attention to the fact that the free field realization of the Macdonald operator is based on the kernel function for the operator. Further, Komori, Noumi, and Shiraishi have introduced the kernel function for the elliptic Ruijsenaars operator. Thus the author considered that the free field realization of the elliptic Ruijsenaars operator should be based on the kernel function introduced by Komori, Noumi, and Shiraishi. In fact, starting from the kernel function for the elliptic Ruijsenaars operator, the author succeeded in constructing the free field realization of the operator. Furthermore, it turned out that the boson operators used in the free field realization satisfy the relations which are elliptic analog of the defining relations of the Ding-Iohara-Miki algebra. Consequently, we obtain an elliptic analog of the Ding-Iohara-Miki algebra, and we call the algebra the elliptic Ding-Iohara-Miki algebra.

Elliptic Ruijsenaars operator —**free field realization**→ **Elliptic Ding-Iohara-Miki algebra**

↑ elliptic deformation ↑ **elliptic deformation**

Macdonald operator —free field realization→ Ding-Iohara-Miki algebra

In the above story, we have used a new free field realization or boson operators whose well-definedness are guaranteed by series expansion in p, where p is a formal variable corresponding to the elliptic modulus.

The author have also checked that by the free field realization of the elliptic Ruijsenaars operator and the elliptic Feigin-Odesskii algebra, we could construct two commuting families of q-difference operators containing the elliptic Ruijsenaars operators.

The (trigonometric) Ding-Iohara-Miki algebra has been applied to some materials in mathematical physics such as the 5-dimensional version of AGT conjecture, the refined topological vertex which is used in the topological string theory. Thus it is probable that the elliptic Ding-Iohara-Miki algebra make a new stream in mathematical physics in a future.